A One-Electron Theory of Nonrelativistic Quantum Temporal Dynamics

Abstract:

We generalize the framework of nonrelativistic quantum mechanics, which treats time as a parameter rather than an operator, by promoting standard time to an operator and, as a consequence, introducing a two-dimensional temporal system. As a result, quantum states over time as well as space are considered, and a quantum theory of temporal dynamics is developed. This framework is used to support a new version of the Feynman one-electron universe hypothesis, by postulating the existence of large potential barriers in the past and future that effect the repeated temporal oscillations of a single particle, thus producing all particles in the universe as a consequence. A simple nonrelativistic, one-electron model is used for this quantum temporal dynamics.

I. Introduction

The Nature of Time: Philosophy and Physics

The nature of time is the greatest unsolved problem in philosophy and theoretical physics. Indeed, temporal progression is even more fundamental to the structure of the universe than spatial extent. The passage of time determines the evolution of all physical systems, and its regularity and continuity are the source of order in the cosmos. Yet, despite the centrality of time in the framework of reality, its ultimate nature remains a mystery. Is time an emergent property of a hidden structure, or is it irreducible? Is the progression of time generally linear, or is it subject to fluctuations and other quantum phenomena? Is travel to the past a possibility? In the quest to answer these deep questions, philosophy is increasingly relying upon theoretical physics and cosmology, whose insights are revealing the structure of the universe. The present paper explores a novel physical theory of time, and the role of quantum mechanics in its passage.

Classical mechanics treats time as an absolute parameter, not as an emergent property of some deeper structure. As such, while it explains the motion of mechanical bodies through space, it assumes a linear, unchanging form of *temporal* motion. This model of time is central to the "clockwork universe" of Newton, which sees the future history of the universe as determined from a set of differential equations and initial conditions[6]. Once the position and momentum of each element of a closed system is known, the future positions of each such element are determined by the mathematical formalism of classical mechanics. Quantum mechanics discards such spatial determinism, modelling particles through wavefunctions over physical space. The formalism of quantum theory replaces spatial determinism with spatial uncertainty, but maintains the absolute progression of time, in its nonrelativistic incarnation. If space is an uncertain feature of the quantum world, should the temporal continuum also be subject to quantum uncertainty?

The framework of special relativity, and the clockwork formalism of classical mechanics, lends credence to the *eternalist* theory of time, that the entirety of the future and past history of the universe exists as a single entity[1]. As such, temporal progression is an illusion of experience, an intrinsic property of a certain observational viewpoint within the spacetime structure, rather than an external characteristic. The probabilistic nature of quantum mechanics, and particularly the uncertainty of quantum state collapse, suggests the *growing block* theory, that only the past and present "exist" as such. It is indeed this phenomenon of state collapse that divides eternalism from quantum mechanics. Should the many-worlds interpretation prove accurate, that wavefunction collapse is an illusion induced by the observer's limitation, eternalism would prove compatible with quantum theory.

Temporal Nonlinearity and Temporal Emergence

The structure of nonrelativistic quantum mechanics implies certain violations to the traditional notion of temporal causality. The "delayed choice quantum eraser" experiment, a variation of the classic "double-slit" apparatus conceived by Marlan Scully, suggests that information from future events can affect the present[20]. Nonetheless, analysis of the thought experiment indicates that such violations of causality are consequences of the random behavior of quantum state collapse to an eigenvector, and that the many-worlds interpretation would prevent such retrocausality phenomena. While the linearity of the state evolution equations preserves the determinism of classical mechanics, the probabilistic nature of eigenvector collapse indeed drives many of the seemingly paradoxical characteristics of quantum systems.

General relativity, alternatively, allows causality violation through closed timelike curves, which allow superluminal communication and therefore retrocausality[12]. While locally the passage of information is never superluminal[18], the curvature of the spacetime continuum allows global phenomena to violate causal relationships and the standard sequence of time[15]. Richard Gott's solution of spacetime near a cosmic string allows a hypothetical traveler to interact with their past self, after travelling near the string[13]. Generally, however, such solutions to the Einstein Field Equations permitting time travel require negative energy densities, which have not been observed in nature[2].

The question of whether closed timelike curves exist depends on the nature of quantum gravity, which continues to elude the theoretical physics community. Different combinations of quantum mechanics and general relativity, including various models of quantum field theory in curved spacetime, suggest different answers to this question[10]. Some theories of quantum gravity, particularly loop quantum gravity, propose that the temporal continuum is an emergent property of some more fundamental structure. LQG proposes that time, although linear on macroscopic scales, is a result of a Planck-scale discontinuous topological structure. Other such models, including twistor theory, also suggest the emergence of time from a Planck-scale system[4]. Indeed, the property of background-independence is a coveted aspect of quantum gravity theories[5].

Novel Theories of Temporal Structure

The cyclical nature of time is a repeated theme in various models of cosmology. Roger Penrose's Conformal Cyclical Cosmology proposes that absolute temporal durations are irrelevant in both the entropic future state of the universe, and the high-energy initial singularity, and as such, that the geometry of the universe at these times is primarily a conformal one[16]. By using this conformality of time, he scales spacetime so as to continuously identify the future entropic history with a big-bang-like beginning, producing repeated cycles of time[17]. A variety of cyclical cosmological models, including the "big splat" model of M-Theory and the "big bounce" theories of Teller, also suggest that temporal progression has a cyclical nature[8].

John Wheeler, in a communication with Richard Feynman, developed a remarkable cyclical model of time, proposing that every electron in the universe is actually the *same* electron, moving backward and forward through time[7]. To an observer at any specific time, whose spatial slice intersects spacetime, it would appear as if multiple electrons are present. Such a model of temporal progression, in which the passage of time itself can reverse directions, suggests a notion of "temporal mechanics," that movement through time can be understood in the same way as motion through space. Such a framework would undoubtedly require additional temporal dimensions, a notion that has been explored in certain models of quantum gravity. M-Theory, the hypothetical framework unifying the different string theories, might involve hidden dimensions of time, compacified on a small scale[3]. If such additional temporal dimensions exist, the passage of time is far more complex a phenomenon than standard theories would suggest.

The mathematical formalism of nonrelativistic quantum mechanics, despite introducing uncertainty in spatial position, nonetheless upholds the absolute motion through time of classical mechanics. While a particle can exist in a superposition of distinct physical locations, its temporal position is absolute, and the probability amplitude extends only over physical space. A full understanding of the progression of time, however, should rest on a physical theory that explains movement through the temporal continuum as a consequence of a dynamical equation, rather than assuming such ad hoc. By allowing quantum dynamics to include temporal progression, and considering wavefunctions over time as well as space, quantum mechanics would see its fullest incarnation. The most natural development of physical law—from spatial certainty to spatial uncertainity—would resolve temporal certainty likewise as a macroscopic consequence of a quantum system with temporal uncertainity. The present paper concerns itself with constructing a quantum theory of temporal dynamics.

II. A Thought Experiment and Its Implications

Let us consider the following hypothetical situation. An apparatus, a "time machine", is capable of temporal travel. When the experimenter activates the apparatus, it "jumps" to a future time, say three years subsequent to the experiment, remains for one hour, and "jumps" back to the time when the experimenter initiated the apparatus. To an observer internal to the apparatus, it appears as if each temporal "jump" takes one hour to complete. The experimenter places a clock in the machine, reading 3:00 AM 9/23/2070, and activates the device.

From the perspective internal to the machine, that of the clock, three hours elapse over the course of the temporal journey. Each jump, both forward and backward, takes one hour to complete, and the machine spends one hour in "normal time" in 2073. From the perspective of an external observer, on 3:00 AM on 9/23/2070, the apparatus vanishes (through some unspecified process it leaves the standard time continuum) and an identical apparatus simultaneously appears in a different location. The experimenter determines that this other machine contains the clock he prepared minutes before. Removing the clock from this machine, he discovers that it reads 6:00 AM 9/23/2070—exactly three hours ahead of its original value.

From the perspective of the experimenter, no time has elapsed between the clock entering "jumpspace" and the clock returning to 3:00 AM, 9/23/2070. Yet, the deviation between this time, and the reading of the clock, is not puzzling to him. He realizes that each jump through time, and the brief period outside "jumpspace" in 2073, requires one hour of duration from the perspective of the clock. As such, a difference exists between the "internal time" of the apparatus and the "external time" of the surrounding system. Relative to the external system, the clock appears at the moment it left. Relative to its internal time, the clock's journey requires three hours.

This thought experiment exemplifies the distinction between the "internal time" or "instrinsic time" of a particle or sub-system, and the "extrinsic time" of the universe or surrounding system. The experimenter, following his test on the clock, enters the apparatus, while an external observer watches. The observer notes, as before, that the experimenter disappears (enclosed within the apparatus) and immediately reappears. Call the initial state of the experimenter (before the jump) State A, and the subsequent state of the experimenter (after the completion of both jumps) State B. From the perspective of the observer, no time has elapsed between State A and State B. Relative to external time, or "realtime", State A and State B exist simultaneously. Yet, during both jumps and the brief interlude in "normalspace", the state of the experimenter has undoubtedly changed. As such, State A and State B, despite existing coincident in realtime, are distinct. This experiment reveals that states evolve relative to "intrinsic time", and not relative to "external time".

The Aboriginal Australians conceived of another dimension of temporal duration[11], a "dreamtime" or "time outside of time". In a fanciful spirit, we shall adopt this nomenclature to describe "intrinsic time". The states of entities evolve relative to dreamtime, but their relation with an external system is defined by realtime. If the backwards jump of the apparatus is greater than the forwards jump, so that State B exits the apparatus before State A enters, the experimenter can interact with his past self, as both exist at the same position in realtime. As such, causality exists relative to realtime, but not relative to dreamtime, for the future and past dreamtime states of an entity can hypothetically interact.

This distinction between "internal" and "external" time provides a framework for understanding the one-electron universe of Wheeler. The ultimate goal of our quantum theory of temporal dynamics is to describe the kinematics of temporal motion, and particularly how the realtime position of a system evolves. Such a formalism would consider the internal time of a particle or system, and determine not only the probability that the particle will be found in some spatial location as a function of time, but also the chance that the particle exists at some realtime position, as a function of dreamtime. We might imagine, for example, that the apparatus is capable of random "jumps" relative to external time, and that we determine the probability that the clock internal to the box is located at any point in external time as a function of its internal time.

Thus, the wavefunction is a function of both spatial position and realtime, changing with dreamtime. Such a "temporal wavefunction" might reflect between two potential barriers, resulting in a motion of the corresponding particle forwards and backwards through realtime. Indeed, in this model, only a single electron need exist, and this dynamical motion through time produces the apparent deluge of electrons at any specific realtime position.

III. Towards a Quantum Theory of Temporal Dynamics

The present paper endeavours to construct a quantum theory of temporal dynamics, describing the chronological "position" of a particle much in the same way that standard quantum theory treats its physical position. Much as typical quantum particles exist in a superposition of distinct physical locations, a continuous "wavefunction" over space, whose value is proportional to the associated probability amplitude, a particle in "temporal quantum theory" would exist in a continuous superposition of distinct *times*, with a nonzero probability of existing in one *time* or another. This project represents the fullest completion of quantum theory, which, while discarding the Newtonian conception of *definite space*, nonetheless upholds that of *definite time*.

It should be noted that the quantum temporal dynamics explored in this article will be nonrelativistic; that is, applying to systems with low energy, where the standard Schrodinger equation serves as a useful approximation. Of course, any complete theory of time would include the effect of relativistic phenomena, but the development of a quantum field theory of temporal dynamics extends far beyond the scope of the present paper. Such an enterprise represents the most natural extension of these results, but its implementation would involve an investigation of considerable complexity. As such, all the succeeding analysis of the theory will start by assuming nonrelativistic quantum theory, and particularly the standard Schrodinger equation and low-energy Hamiltonian. Nonetheless, some aspects of special relativity, and particularly the form of the Minkowski metric, will be considered.

The centerpiece of QTD is an equation describing the evolution of "temporal wavefunctions", which take as argument spacetime position and yield the probability ampitude (or weighting coefficient in the superposition) that the particle will be found in that state. Clearly, as the thought experiment of the preceeding section demonstrated, an additional variable must be included, with respect to which these temporal wavefunctions evolve. This variable, which we called "dreamtime", could be considered a sort of "intrinsic time" relative to the particle, rather than the "extrinsic time" of the larger system (or universe). By providing a mathematical mechanism for describing how temporal states evolve, we not only make the remarkable proposition that particles can exist in a superposition of different times, but also that "movement through time" is a mechanical quality. That is, we propose that the movement of systems through time can be analyzed in much the same way as their movement through physical space, and that this movement is subject to quantum phenomena (e.g., fluctuations and "jumps"). Much as quantum theory reduces to the standard classical formalism for macroscopic systems, our QTD would reduce to the standard, linear progression through time for macroscopic phenomena. However, on the Planck scale, nontrivial "temporal mechanics" might exist.

As we shall consider in Section VII, QTD lends credence to the "one-electron" hypothesis of John Wheeler, by suggesting that every electron in the universe (or quark, muon, etc.) is actually the *same* electron, but moving back and forth through time, so to the observer at any *specific* time, it appears as if a multitude of electrons are present. The original one-electron universe was proposed in the context of quantum field theory, but QTD instead employs nonrelativistic quantum mechanics, providing a mechanism for this electron to move through time. Roughly, QTD suggests that vast potential barriers exist at the past and future singularities of the universe, and that *the* electron bounces back and forth between these barriers, populating the cosmos. Eventually, the electron might tunnel through one of the barriers, and populate another universe; in Section VIII, this remarkable cyclical cosmology will be investigated.

To develop the central equation of QTD, we begin by analyzing the standard one-dimensional Schrodinger equation for a single particle:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (3.1)$$

In QTD, we replace the standard wavefunction $\psi(\boldsymbol{r},t)$ with a more general temporal form $\psi(\boldsymbol{r},t;T)$, where t is the external "realtime" and T is the "dreamtime" coordinate with respect to which ψ evolves. As such, ψ is primarily a function of $(\boldsymbol{r},t)$, expressing the probability amplitude that the particle will be found at $(\boldsymbol{r},t)$, and the form of this function changes with dreamtime T. The most natural way to generalize (3.1) to the case of a temporal wavefunction $\psi(\boldsymbol{r},t;T)$ is to treat $(\boldsymbol{r},t)$ itself as a spatial coordinate, as it serves an analogous purpose as $\boldsymbol{r} = \{x,y,z\}$ in standard quantum mechanics, and treat T, the parameter effecting wavefunction evolution, as the temporal variable. Now, the del-squared (Laplacian) operator on the Minkowski space of four-dimensional spacetime $(\boldsymbol{r},t)$ is the d'Alembertian, given as

$$\Box = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \quad (3.2)$$

With c the speed of light in vacuum; generalizing (3.1), replacing the spatial coordinates with the four-dimensional $(\boldsymbol{r}, t)$ and time by T, we have

$$-\frac{\hbar^2}{2m}\Box\psi + V\psi = i\hbar\frac{\partial\psi}{\partial T} \quad (3.3)$$

This represents a "first attempt" at creating a consistent mathematical equation for the evolution of Dreamtime states. Unfortunately, as Section VI describes, equation (3.3) is inconsistent with the standard Schrodinger equation, and would produce anomalies (like temporal accelerations) that have not been observed in nature. In order to achieve consistency, (3.3) must be modified by adjusting the formula for the d'Alembertian operator. Specifically, we must replace the speed of light c with a free parameter a, producing the "modified d'Alembertian"

$$\Box_a = \nabla^2 - \frac{1}{a^2}\frac{\partial^2}{\partial t^2} \quad (3.4)$$

The value of a, the "temporal velocity", is unspecified by the theory itself, but Section VI estimates its value given experimental data. Thus, our modified quantum equation is

$$-\frac{\hbar^2}{2m}\Box_a\psi(\boldsymbol{r}, t; T) + V\psi(\boldsymbol{r}, t; T) = i\hbar\frac{\partial\psi(\boldsymbol{r}, t; T)}{\partial T} \quad (3.5)$$

describing the evolution of temporal quantum states in dreamtime T. The natural next step is to consider the form of the potential V.

IV. The Potential in Quantum Temporal Dynamics

The Underlying Potential Formula

Typical notions of causality do not apply in QTD. Previous attempts to construct theories involving multiple time dimensions determined that interactions between entities are only possible if both entites exist at the same two-time coordinate $(t; T)$. Quantum Temporal Dynamics, although a theory involving two temporal dimensions (dreamtime and realtime) is completely different, as it treats these dimensions in a distinct fashion. Realtime is incorporated into the d'Alembertian operator with the spatial coordinates, while dreamtime has the same role as standard time in the traditional Schrodinger equation. Realtime is an external coordinate indexing the position of a particle relative to the larger system, while dreamtime evolves the state of that particle. As our thought experiment demonstrated, causality exists relative to realtime, not dreamtime.

If a particle's temporal wavefunction evolves in T so that it returns to a position t that it occupied at an earlier value of T, to the external observer at t, it appears as if two particles are present. These "particles" can clearly interact, e.g. through electrostatic forces. As the previous section demonstrated, self-interaction is possible, if a particle's future and past dreamtime selves coincide at the same position in realtime. As such, realtime indexes causal positions, while temporal evolution itself is effected through dreamtime. If dreamtime and realtime do not coincide, apparent violations of causality can occur.

Such a model explains the one-electron hypothesis, as the evolution of the single electron in dreamtime (reflecting between two potential walls) produces an apparent deluge of electrons at any realtime position t, being the incarnations of the particle at distinct dreamtime positions. Despite existing at different dreamtime points, these particles clearly can interact. In the remainder of the present article, we shall consider a nonrelativistic universe consisting only of electrons, and analyze how QTD can explain the characteristics of this model universe. Further development of QTD should consider interactions between distinct particle types (e.g. quarks, leptons, etc.) in a relativistic framework.

We open our analysis of this one-electron universe by considering the form of the potential V. The potential consists of two central components, the enormous potential walls at the beginning and end of the universe, and the electrostatic interactions (Coulomb force) between the consistuent particles of the universe (we simplify the model by neglecting other contributions to the potential). The former is clearly a function of the form $R(t)$ of realtime, being zero for most of realtime and adopting a high value for two values of realtime. The nature of this function will be explained in Section VII. The particle interactions constitute the self-interaction of the electron with itself, as its states at different dreamtimes conincide in realtime. Denoting the self-interaction by $S(\boldsymbol{r},t)$, we have that the potential is given as

$$R(t) + S(\boldsymbol{r},t) \quad (4.1)$$

Now we consider the form of the self-interaction $S(\boldsymbol{r},t)$. Clearly, this potential is a sum over contributions from states of the electron at distinct dreamtimes, that coincide in realtime. The obvious form of $S(\boldsymbol{r},t)$ is an integral over T', of the potential at $(\boldsymbol{r},t)$

generated by the electron at T' (whose state is $\psi(\boldsymbol{r}', t'; T')$). That integrand itself is an integral, over $(\boldsymbol{r}', t')$, of the contribution to the potential at $(\boldsymbol{r}, t)$ by the portion of the wavefunction at $(\boldsymbol{r}', t')$. The most natural way to derive this contribution is to consider the probability amplitude itself as a charge density, and thus multiply by $|\psi(\boldsymbol{r}', t'; T')|^2$.

This contribution is proportional to $V(\boldsymbol{r} - \boldsymbol{r}')$, the potential between charges at $\boldsymbol{r}$ and $\boldsymbol{r}'$. Clearly, states significantly separated in realtime do not interact, but as the temporal wavefunctions will have some uncertainty ("fuzziness") in realtime position, we should allow for some slight violations in realtime causality. We account for such by multiplying the above contribution by $\delta_\varepsilon(t - t')$, a Gaussian with width ε that highly weights contributions from states close in realtime. Considering the above argument, we modify (4.1) to produce the formula

$$R(t) + \iiint |\psi(\boldsymbol{r}', t'; T')|^2 V(\boldsymbol{r} - \boldsymbol{r}') \delta_\varepsilon (t - t') d\boldsymbol{r}' dt' dT' \quad (4.2)$$

for the potential at $(\boldsymbol{r}, t)$. We imagine that the width of typical temporal waveforms in realtime is on the Planck scale, a level unobservable by present technology, and as such, that the "fuzziness" of these quantum states would presently go unnoticed. We make this proposal, as the Planck scale seems to be the level at which novel physical phenomena emerge. Thus, the scale of ε, the width of the Gaussian that sets the level of realtime causality violations, is roughly the Planck time.

To prevent particles from interacting with their present dreamtime state, we stipulate that the outermost integral over dreamtime should exclude some small interval $(T - \lambda, T + \lambda)$, with λ on the scale of the Planck time. $V(\boldsymbol{r} - \boldsymbol{r}')$ itself is a form of potential "density", with different units than the standard Coulomb potential. $R(t)$, explained in Section VII, constitutes the enormous potential walls at the past and future singularities of the universe. (4.2) is the foundation of QTD's analysis of the potential.

Deriving the Known Formula

Now that the general form of the potential in QTD, (4.2), is known, it seems natural to make sense of it, and how known physical phenomena emerge from it. We begin by analyzing the formula for the self-interaction:

$$\iiint |\psi(\boldsymbol{r}', t'; T')|^2 V(\boldsymbol{r} - \boldsymbol{r}') \delta_\varepsilon (t - t') d\boldsymbol{r}' dt' dT' \quad (4.3)$$

We write the wavefunction as a separation of spatial and temporal states, $\psi(\boldsymbol{r}', t'; T') = \theta(\boldsymbol{r}', T')\phi(t', T')$, with θ adopting the role of the standard quantum wavefunction (the probability amplitude commonly measured) and $\phi(t', T')$ a Gaussian with Planck-scale width that generally moves forward in realtime as dreamtime progresses (a wavepacket; see the succeeding section for a more thorough description). ϕ reflects against the enormous potential barriers $R(t)$, thus producing the back-and-forth motion of the electron through dreamtime. This temporal wavepacket might be moving forward or backward relative to realtime, depending on what leg of the cosmic journey the electron is currently on. We set $t'_{T'}$ to be the center of this Gaussian, so that for $t' \neq t'_{T'}$, $\phi(t', T')$ rapidly decays on Planck-level scales of $t'_{T'} - t$. Introducing this separation into the above formula, we have

$$\iiint |\theta(\boldsymbol{r}', T')|^2 |\phi(t', T')|^2 V(\boldsymbol{r} - \boldsymbol{r}')\delta_\varepsilon(t - t')d\boldsymbol{r}'dt'dT'$$

Now, δ_ε decays quickly for $t' \neq t$. Thus, only those values of T' such that $t'_{T'} \approx t$ contribute significantly to the integral. Let us define $T_j{}'$ so that $t'_{T_j{}'} = t$. Thus, the integral over dreamtime reduces, approximately, to a sum of smaller integrals about each separate $T_j{}'$ (excluding, of course, T itself). Thus, we have

$$\sum_j \iiint |\theta(\boldsymbol{r}', T')|^2 |\phi(t', T')|^2 V(\boldsymbol{r} - \boldsymbol{r}')\delta_\varepsilon(t - t')d\boldsymbol{r}'dt'dT'$$

which, employing the separation of variables, reduces to

$$\sum_j \int \left(\int |\theta(\boldsymbol{r}', T')|^2 V(\boldsymbol{r} - \boldsymbol{r}')d\boldsymbol{r}'\right)\left(\int |\phi(t', T')|^2 \delta_\varepsilon(t - t')\, dt'\right) dT'$$

Now, $\theta(\boldsymbol{r}', T')$ likely does not change significantly for small (Planck-order) changes in T'. Indeed, θ represents that standard (spatial) quantum wavefunction that typically measured, and such states usually change over the scale of atomic time, which is orders of magnitude greater than Planck time. As each dreamtime integral in the sum exists over a (roughly) Planck-scale interval, we can treat θ as being effectively constant with respect to T'. We thus have

$$\sum_j \left(\int \left|\theta(\boldsymbol{r}', T_j{}')\right|^2 V(\boldsymbol{r} - \boldsymbol{r}')d\boldsymbol{r}'\right) \int \int |\phi(t', T')|^2 \delta_\varepsilon(t - t')\, dt'\, dT'$$

Assuming that the width of the temporal wavepacket does not change appreciably for significant spans of dreamtime, as Section VI will demonstrate, the temporal integral above (the second factor in the summand) is largely constant, not depending on the value of realtime t. Assuming that the temporal wavepacket's associated probability density is of the form

$$|\phi(t',T')|^2 = exp(-a(t' - t'_{T'})^2) \quad (4.4)$$

a Gaussian of largely unchanging width, the integral in question can be written as

$$\int\int exp(-a(t' - t'_{T'})^2)\, \delta_\varepsilon(t - t')\, dt'\, dT'$$

with the dreamtime integral over a small interval about $T_j{}'$, and the realtime one over a Planck-level interval about t. Simplifying, we have

$$\int\int exp(-a(t' - t - (t'_{T'} - t))^2)\, \delta_\varepsilon(-(t' - t))\, dt'\, dT'$$

It should be noted that, for any particular value of j, the center of the temporal Gaussian can be written as $t'_{T'} = \pm T' + b_j$, with b_j a constant depending on the leg of the electron's temporal journey, and the positive or negative sign depending on the direction of temporal progression. Employing a substitution of variables, we have

$$\int\int exp(-a(w - l)^2)\, \delta_\varepsilon(w)\, dw\, dl$$

a formula that doesen't depend on the value of realtime t. Each integral is over a small, Planck-sized interval about zero. Referring to the above integral as the constant p, the approximate formula for the self-interaction reduces to

$$\sum_j p\left(\int \left|\theta(\boldsymbol{r}', T_j{}')\right|^2 V(\boldsymbol{r} - \boldsymbol{r}') d\boldsymbol{r}'\right)$$

Incorporating a value of the potential density $V(\boldsymbol{r} - \boldsymbol{r}')$ proportional to the standard Coulomb factor, we have

$$\sum_j p\left(\int \left|\theta(\boldsymbol{r}', T_j{}')\right|^2 \frac{Dq^2}{|\boldsymbol{r} - \boldsymbol{r}'|} d\boldsymbol{r}'\right)$$

where q is the charge on the electron, and the constant D is proportional to the Coulomb constant k as

$$D = k/p$$

with p given as

$$p = \int\int exp(-a(w-l)^2)\ \delta_\varepsilon(w)\ dw\ dl \quad (4.5)$$

where the Gaussian is

$$\delta_\varepsilon(x) = \frac{1}{\varepsilon\sqrt{2\pi}} exp\left(-\frac{x^2}{2\varepsilon^2}\right) \quad (4.6)$$

Given this value for D, the self-interaction is approximately

$$S(\boldsymbol{r},t) = \sum_j \int \left|\theta(\boldsymbol{r}',T_j')\right|^2 \frac{kq^2}{|\boldsymbol{r}-\boldsymbol{r}'|} d\boldsymbol{r}' \quad (4.7)$$

with T_j' defined so that $t'_{T_j'} = t$. This formula yields the potential at $(\boldsymbol{r},t)$ for a nonrelativistic, one-electron universe where electrostatic force communication is instantaneous.

Comparing With the Known Formula

Equation (4.7), which is only an approximation of (4.3), reduces to the well-known formula for the potential. To the perspective of the observer affixed to a particular point in realtime, the subject of our thought experiment, it appears as if the contribution of each different j in the sum of (4.7) derives from a separate particle. In reality, these correspond to distinct points in dreamtime of the same particle; this is simply a restatement of the notion that causality is violated in dreamtime, allowing a particle's future dreamtime self to interact with (affect) its past dreamtime self. In the one-electron universe, it is indeed this self-interaction that is responsible for all particle forces, aside from the potential walls at the ends of the universe.

Indeed, (4.7) is simply the formula for an electrostatic potential caused by a number of different particles indexed by j, each particle having the wavefunction $\theta(\boldsymbol{r}',T_j')$. Furthermore, as is well-known, the standard spatial wavefunction of a particle can be treated electrostatically as a charge density, with the potential caused by the particle an integral of the form

$$\int |\psi(\boldsymbol{r}',t)|^2 \frac{kq^2}{|\boldsymbol{r}-\boldsymbol{r}'|} d\boldsymbol{r}'$$

where the squared modulus of the wavefunction acts as charge density[9]. As such, (4.7) gives us a potential caused apparently by multiple different particles, each of which is the single electron on a different lap of its back-and-forth journey through time. Of course, however, this equation is only an approximation, assuming, for example, that the temporal wavepacket decays to virtually zero for values of realtime significantly different from the center ${t'}_{T'}$. In reality, there might be negligably small deviations from (4.7), particularly on Planck scales.

V. Derivation of the Decoupled Equations and Temporal Duality

Given (4.2), our temporal Schrodinger equation, (3.5), becomes

$$-\frac{\hbar^2}{2m}\Box_a\psi(\boldsymbol{r},t;T)+\left\{R(t)+\iiint|\psi(\boldsymbol{r}',t';T')|^2V(\boldsymbol{r}-\boldsymbol{r}')\delta_\varepsilon(t-t')d\boldsymbol{r}'dt'dT'\right\}\psi(\boldsymbol{r},t;T)$$
$$=i\hbar\frac{\partial\psi(\boldsymbol{r},t;T)}{\partial T}\quad(5.1)$$

Equation (5.1) is the central equation of Quantum Temporal Dynamics. All the aspects of the theory, besides the precise form of the endtimes potentials explored in Sections VII and VIII, can be derived from this equation. Let us begin by considering the solutions separated in realtime and space that were introduced in the previous section. That is, we set

$$\psi(\boldsymbol{r},t;T)=\theta(\boldsymbol{r},T)\phi(t,T)\quad(5.2)$$

describing the quantum state as a product of the spatial wavefunction (the state considered in standard quantum mechanics) and the temporal wavepacket, which typically moves forward in a linear fashion. For the time being, we simplify our analysis of the previous section, and consider a potential given as

$$V=V(\boldsymbol{r})+R(t)\quad(5.3)$$

the sum of an unchanging spatial potential and the endtimes potential. (5.3) constitutes a dramatic simplification of (4.7), where the electrostatic potential is modelled as unchanging in realtime. With this simplified potential, subsituting the separated form of the wavefunction, we have

$$-\frac{\hbar^2}{2m}\Box_a\theta(\boldsymbol{r},T)\phi(t,T)+\{R(t)+V(\boldsymbol{r})\}\theta(\boldsymbol{r},T)\phi(t,T)=i\hbar\frac{\partial}{\partial T}\{\theta(\boldsymbol{r},T)\phi(t,T)\}$$

Simplifying, and applying the definition of $\Box_a$, results in

$$-\frac{\hbar^2}{2m}\left(\nabla^2 - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}\right)\theta(\boldsymbol{r},T)\phi(t,T) + \{R(t) + V(\boldsymbol{r})\}\theta(\boldsymbol{r},T)\phi(t,T)$$
$$= i\hbar\{\theta^*(\boldsymbol{r},T)\phi(t,T) + \theta(\boldsymbol{r},T)\phi^*(t,T)\}$$

This yields

$$-\frac{\hbar^2}{2m}\left\{\phi(t,T)\nabla^2\theta(\boldsymbol{r},T) - \theta(\boldsymbol{r},T)\frac{\partial^2}{\partial t^2}\frac{1}{a^2}\phi(t,T)\right\} + \{R(t) + V(\boldsymbol{r})\}\theta(\boldsymbol{r},T)\phi(t,T)$$
$$= i\hbar\{\theta^*(\boldsymbol{r},T)\phi(t,T) + \theta(\boldsymbol{r},T)\phi^*(t,T)\}$$

Dividing through by $\theta(\boldsymbol{r},T)\phi(t,T)$, we have

$$-\frac{\hbar^2}{2m}\left\{\frac{\nabla^2\theta(\boldsymbol{r},T)}{\theta(\boldsymbol{r},T)} - \frac{1}{a^2}\frac{\phi''(t,T)}{\phi(t,T)}\right\} + \{R(t) + V(\boldsymbol{r})\} = i\hbar\left\{\frac{\theta^*(\boldsymbol{r},T)}{\theta(\boldsymbol{r},T)} + \frac{\phi^*(t,T)}{\phi(t,T)}\right\}$$

Simplifying,

$$-\frac{\hbar^2}{2m}\frac{\nabla^2\theta(\boldsymbol{r},T)}{\theta(\boldsymbol{r},T)} - i\hbar\frac{\theta^*(\boldsymbol{r},T)}{\theta(\boldsymbol{r},T)} + V(\boldsymbol{r}) = -\frac{\hbar^2}{2ma^2}\frac{\phi''(t,T)}{\phi(t,T)} + i\hbar\frac{\phi^*(t,T)}{\phi(t,T)} - R(t) = D(T)$$

As functions of space and realtime are equal to each other, the only possibility is that both are equal to a constant $D(T)$ (which itself generally depends on dreamtime, but not on $(\boldsymbol{r},T)$). Reducing the resultant couplet of equations, we have

$$-\frac{\hbar^2}{2m}\nabla^2\theta(\boldsymbol{r},T) + (V(\boldsymbol{r}) - D)\theta(\boldsymbol{r},T) = i\hbar\frac{\partial}{\partial T}\theta(\boldsymbol{r},T)$$
$$-\frac{\hbar^2}{2ma^2}\frac{\partial^2}{\partial t^2}\phi(t,T) + (-R(t) - D)\phi(t,T) = -i\hbar\frac{\partial}{\partial T}\phi(t,T)$$

Now, each resultant equation is a variation of the standard Schrodinger equation (3.1), in the latter case with a modified mass $-ma^2$. As is well known, addition of any time-varying potential that is constant over space produces equivalent solutions to the Shcrodinger equation, perhaps with an added phase. Indeed, a potential constant over space produces no forces, and therefore is physically indistinguishable from no potential at all. Thus, the two equations above are *equivalent* to

$$-\frac{\hbar^2}{2m}\nabla^2\theta(\boldsymbol{r},T) + V(\boldsymbol{r})\theta(\boldsymbol{r},T) = i\hbar\frac{\partial}{\partial T}\theta(\boldsymbol{r},T) \quad (5.4)$$

$$\frac{\hbar^2}{2ma^2}\frac{\partial^2}{\partial t^2}\phi(t,T) + R(t)\phi(t,T) = i\hbar\frac{\partial}{\partial T}\phi(t,T) \quad (5.5)$$

(5.4) is the standard Schrodinger equation, but with realtime replaced with dreamtime. As such, it stipulates that the spatial waveform evolves in dreamtime the same way that standard wavefunctions evolve in time. (5.5) describes the temporal wavepacket $\phi(t,T)$, which follows the typical Schrodinger equation, but with the potential given as $R(t)$ and the mass as $-ma^2$. We imagine, as was noted in the previous section, that this wavepacket is an extremely thin Gaussian (roughly Planck-width) that moves forward with a unitless speed of 1. This represents the typical progression through time, and when the wavepacket moves in such a way, for all intents and purposes dreamtime and realtime are equivalent, thus reducing (5.4) to the standard equation in realtime. However, at the points in realtime where the endtimes potential $R(t)$ becomes relevant, the temporal wavepacket reflects and reverses direction.

(5.5) solidifies a notion called *temporal duality*. Namely, when atomic units are employed, the temporal evolution of a system is equivalent to the spatial evolution of a system, with the mass multiplied by $-a^2$. In this case, a speed of 1 in the temporal situation corresponds to a speed of roughly (10^{-10} m) / (10^{-17} sec) in the spatial situation, etc. Thus, we need only consider the modified spatial system to determine the evolution of the temporal system.

Conveniently, the probability density of the wavefunction is the product of the densities of the spatial and temporal wavefunctions, as

$$\iint |\psi|^2 \, d\boldsymbol{r}dt = \iint |\phi(t,T)|^2 |\theta(\boldsymbol{r},T)|^2 \, d\boldsymbol{r}dt$$

$$= \left(\int |\phi(t,T)|^2 dt\right)\left(\int |\theta(\boldsymbol{r},T)|^2 \, d\boldsymbol{r}\right) \quad (5.6)$$

Thus, normalization of each such function separately ensures normalization of the combined wavefunction.

VI. Derivation of the Temporal Velocity

Now we have developed the mathematical machinery to calculate the temporal velocity a, at least within an order of magnitude. We assume, as before, that the temporal wavepacket $\phi(t,T)$ has a roughly Planck-order width, that remains Planck-order for significant spans of dreamtime T. Quantum wavepackets, in accordance with the uncertainty principle, generally disperse over long periods of time. Limiting this dispersion will provide a lower bound on the value of the temporal velocity. Ultimately, with a value for a high enough, the constant speed of the wavepacket can be maintained, even in the presence of time-varying potentials that would cause significant "temporal acceleration" (changes in the speed of the temporal wavepacket) for low values of a.

We begin by employing the "temporal duality" developed in the previous section, where a temporal system is equivalent to a spatial system (in atomic units). Given the separated temporal equation in a realtime region without the endtimes potentials $R(t)$,

$$\frac{\hbar^2}{2ma^2}\frac{\partial^2}{\partial t^2}\phi(t,T) = i\hbar\frac{\partial}{\partial T}\phi(t,T) \quad (6.1)$$

in atomic units we have the moving wavepacket solution[19]

$$\phi(t,T) = \left(\frac{2c}{\pi}\right)^{1/4}\frac{exp\left(\frac{-ct^2 + i(lt + l^2T/2ma^2)}{1-2icT/ma^2}\right)}{\sqrt{1-2icT/ma^2}} \quad (6.2)$$

where c is a free parameter and

$$\frac{-l}{ma^2} = v$$

The resultant probability density is given as

$$|\phi(t',T')|^2 = \sqrt{2/\pi}\, w\, exp(-2w^2(t - lT/ma^2)^2)$$

or alternatively

$$|\phi(t',T')|^2 = \sqrt{2/\pi}\, w\, exp(-2w^2(t - vT)^2) \quad (6.3)$$

where

$$w = \left(\frac{c}{1+(2cT/ma^2)^2}\right)^{1/2}$$

The width of the probability density Gaussian is proportional to

$$\frac{1}{w}$$

Thus, the width after a dreamtime T is given by

$$\left(\frac{1+(2(1/d)^2T/ma^2)^2}{(1/d)^2}\right)^{1/2} \quad (6.4)$$

where d is the initial width at dreamtime $T = 0$. Our primary concern is that the wavepacket should preserve its Planck-order width over its entire journey. As there are

10^{80} particles in the observable universe, we can infer that the electron makes at least 10^{80} laps between the two potential reflectors $R(t)$. Assuming that each such lap is on the order of 10^{10} years in duration, in atomic units, the total length of the journey in dreamtime is 10^{114} (the length in realtime is on the order of 10^{17}), before the electron possibly tunnels through one of the potential barriers (see Section VII for an analysis of this phenomenon). Thus, the dispersion of the temporal wavepacket over this journey is

$$10^{114}/mda^2$$

with d the initial width. As the Planck time is 10^{-26} in atomic units, and the electron's mass is 1, our requirement reduces to

$$10^{114}/(a^2 10^{-26}) \approx 10^{-26}$$

giving the "effective temporal mass"

$$a^2 = 10^{166} \quad (6.5)$$

in atomic units, yielding the temporal velocity

$$a \approx 10^{81}\, c \quad (6.6)$$

Alternatively, the uncertainty principle applying in the dual spatial system

$$(\Delta x)^2(\Delta p)^2 \geq \frac{\hbar^2}{4}$$

provides the same limits on the size of a, for in atomic units we have

$$(\Delta x)^2(a^2\Delta v)^2 \geq \frac{1}{4} \quad (6.7)$$

where the temporal wavepacket dispersion is given by ΔvT. As the corresponding spatial uncertainty $\Delta x = 10^{-26}$, we have that

$$\Delta vT \geq \frac{1}{2}a^{-2}10^{140}$$

applying the value $T = 10^{114}$. As this quantity should remain on the order of 10^{-26}, a should be on the order of $10^{81}\, c$.

For such a large effective mass in (5.5), any anomalous changes in the speed of the temporal wavepacket, due to time-varying potentials that appear in the equation, will be negligable. Even if the electron's temporal wavepacket is subject to a constant force of 1 Newton for its entire existence, temporal duality demonstrates that during this length of dreamtime, the speed of the wavepacket will change by a factor of

$$\frac{\left(\frac{1\,N}{10^{166}\,m_e}\,\frac{1}{10^{17}}\,s\;10^{114}\right)}{0.01\,c} \approx 10^{-46}$$

Thus, it would take enormous time-varying potentials to cause significant changes in the speed of the electron through time. As such, temporal progression is almost always exactly linear, except at the two endtimes potentials, which are large enough (and time-varying) to accelerate the temporal wavepacket.

VII. The Endtimes Potential

Quantum Temporal Dynamics describes the mechanics of how particles move through time. As such, it permits not only the linear progression through time, but also accelerations, reflections, and other phenomena, claiming that forces and potentials can affect the passage of time; furthermore, quantum phenomena at the Planck scale can cause fluctuations or aberrations in time's passage. Nonetheless, due to the high value for the effective temporal mass (or the temporal velocity) these effects are extremely small, especially at macroscopic scales. We propose, however, that at the past and future singularities of the universe (the Big Bang and, perhaps, the Big Rip, or the conformally modified heat death of Roger Penrose's CCC), vast potentials fill the cosmos, resulting in a time-varying $R(t)$ that is zero for most of realtime and spikes to a high value at two values of realtime. These potentials are rectangular barriers, with an extremely thin base. The repeated back-and-forth motion of the electron between the barriers produces the apparent multitude of electrons at any point in realtime.

As the subsequent analysis will reveal, it appears most natural to set the width of these barriers as inversely proportional to the "effective temporal energy"

$$L = \frac{\hbar}{|E|} \quad (7.1)$$

where E is the kinetic energy of the dual spatial wavepacket. We have that, roughly, $E = \frac{-1}{2}ma^2v^2$, where v is the unitless speed of the wavefunction through time, almost exactly 1. Thus, given the vast effective energy of the temporal wavepacket, the potential barriers are extremely thin. Formula (7.1) was chosen to allow a significant range in the size of the potential barrier to yield roughly the same probability of transmission.

We take the dual spatial situation in atomic units, with a particle of mass $m = -10^{166}$ encountering a rectangular barrier of height U. The probability of transmission (tunneling) is given as[14]

$$exp\left(-2\sqrt{m^2}L\sqrt{\frac{U}{E}-1}\right) \quad (7.2)$$

simplifying to

$$exp\left(-2 * 10^{166}L\sqrt{\frac{U}{E}-1}\right)$$

As the preceding section argued, the probability of transmission each time the electron encounters the future or past barrier is roughly 10^{-80}, as 10^{80} electrons are visible to an observer at any point in realtime. Thus, we have

$$exp\left(-4\sqrt{\frac{U}{E}-1}\right) = 10^{-80}$$

yielding a potential barrier of height

$$U = -2.2 * 10^{169} \quad (7.3)$$

in atomic units, or Hartrees. Alternatively, the barrier is roughly -10^{152} Joules in height.

VIII. A Cyclical Multiverse Through Quantum Tunneling

The preceeding section offers an explanation for the one-electron universe, namely that large potential barriers reflect a particle between two values of realtime, and that roughly 10^{80} laps between these temporal positions produce all the electrons observable at any point in time. As the future and past dreamtime states of a particle can interact electrostatically if they coincide in realtime, as equation (4.7) demonstrates, these seemingly different electrons can interact, producing an apparent multi-particle universe. While 10^{90} years of dreamtime elapse before this universe is "formed" through the back-and-forth motion of the single electron, only 10^{10} years of realtime elapse between the potential barriers.

The analysis of the previous section suggests an intriguing possibility. With the height of the potential barrier we ascertained, a quantum tunneling event through either the past or future barrier is likely after 10^{80} reflections. As such, once the electron finishes populating this universe, it might tunnel through the future potential wall and populate another universe. There might be an infinite, or very large, number of potential barriers, and any interval between two such barriers would constitute a "universe". Quantum tunneling would allow the electron to penetrate from one such "universe" to another.

The calculations of Sections IV and VI assume, however, that the temporal wavepacket evolves over 10^{90} years of dreamtime; if each of the infinite number of barriers has the same height and width, then tunneling through either the past or future barrier of any given universe is equally likely, and hence the electron, after leaving our universe, would eventually tunnel back into it. Thus, the electrons in our universe would correspond to any number of distinct "classes", corresponding to disconnected ranges of dreamtime during which the electron exists in our universe. The total length of dreamtime when the electron exists, in this scenario, is very likely greater than 10^{90} years. To prevent this from happening, it seems natural to allow the height of each successive barrier to be significantly lower than the height of the previous barrier (or vice versa), to promote unidirectional movement through the realtime continuum. Furthermore, the number of universes should be large but finite, and surrounding the multiverse on the past and future realtime sides is an infinite void, through which the single electron travels indefinitely (see **Fig. 8.1**).

As each sucessive barrier is slightly lower than the previous, with a greater probability of transmission for the incident electron, universes to the future in realtime are significantly less dense than universes to the past in realtime, with a lower population of electrons. Further in the past, universes have an ever greater particle density, while further in the future, this density is ever sparser. Ultimately, beyond the past barrier of the multiverse is a single electron which tunnels through the first potential, and beyond the future potential is likewise an endless void with a single electron.

IX. Philosophical Implications

Quantum Temporal Dynamics addresses the problem of time by subjecting temporal progression to the same kinematical formalism that describes movement thorough space. Specifically, it applies the nonrelativistic framework of quantum mechanics, which involves spatial uncertainty, to time, determining the probability that a particle exists in one time or another (and evolving states of temporal superposition). To ensure the consistency of QTD, it is necessary to consider two distinct dimensions of time: the "internal time" of a single particle or sub-system – "dreamtime" – and the "external time" or "realtime" of the surrounding universe. QTD ascertains the probability that a particle exists at any given realtime, as dreamtime moves forward.

As such, the two-dimensionality of time is central to the inner workings of QTD. Temporal Dynamics implies that time, far from being the linear entity of classical mechanics, is an intricate, multi-layered beast, with distinct dimensions and components. Traditional causal relationships exist relative to realtime, as two particles must exist coincident in realtime to interact (assuming instantaneous force transmission); the quantum state of a particle, however, evolves relative to dreamtime. This framework significantly changes the meaning of temporal duration, as the two-fold nature of time (internal and system-wide) is fundamentally different from unidirectional progression.

QTD thus changes the meaning of the problem of time. Realtime suggests an eternalist perspective, namely that the entire history of realtime is pre-determined. Indeed, as the previous section suggested, the values of the potential barriers, even in the realtime future of the universe, are pre-determined (one could say that the "structure" of this multiverse is eternalist, in that it "exists" independent of any particular position in time). The perspective from dreamtime, however, suggests a "growing block" model, in that the evolution of the electron's quantum state is determined relative to dreamtime. However, the form of (4.2) upholds an eternalist framework, as the entire future history of this quantum state must be determined to consider the value of the potential at some point in realtime and some spatial location. Thus, QTD is by nature a fundamentally determinist enterprise; as the randomness of quantum state collapse disrupts such determinism, it seems necessary to unite QTD with the many-worlds intepretation.

The source of time's unexplained nature in the frameworks of theoretical physics is the assumption of unidirectional temporal progression, without consideration of temporal "kinematics". By explaining the progression of time in the same way as mechanics explains motion through space, Quantum Temporal Dynamics provides a physical understanding of time's passage. The reflections of a single particle against potential barriers explain the appearance of matter and energy in the universe. Although a speculative idea, QTD has the potential to change our understanding of the universe.

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