A Path-Integral Formulation of Nonrelativistic Quantum Temporal Dynamics, and Implications on the Multiphase Discretization Metaverse Model

Abstract:

The present paper extends previous work developed by us in [4], [5], where a generalization of path-integration to a complex time-discretization index, suggesting the existence of a certain multiverse, was proposed, and time in nonrelativistic quantum mechanics was promoted from parameter to operator, resulting in a two-time model supporting Wheeler's well-known notion of a one-electron universe. In this article, we develop this quantum temporal dynamics from the path-integral formulation, reducing the foundations of the model to a few simple axioms. Using this formulation, we combine quantum temporal dynamics with the generalized path-integration, in the process providing a cosmological explanation for the location of the parallel universes developed in [5]. We comment on the effects upon temporal flow induced by the alternate time-discretization complex phases in quantum temporal dynamics, describing how superluminal travel and communication might be possible through the alternate universes of this model.

I. Introduction

A. Overview of the Multiphase Discretization Metaverse Model

In [5] the authors considered a generalization of the standard time-discretization framework of quantum mechanics, by generalizing the time-discretization index to arbitrary complex phase. As a result, the limit defining the value of the path-integral, given as

$$P(a,b) \cong \lim_{n\to\infty} \iiint \dots \int \quad exp\left(\frac{i}{\hbar}S\right) dx_1 \dots dx_{n-1} \quad (1.1)$$

has a spectrum of different results when $n \in \mathbb{N} \to z \in \mathbb{C}$, depending on how z approaches infinity in the complex plane. Considering that (1.1), with complex index, represents the most general version of Feynman's path-integral, it is reasonable to suppose that these alternate values for the propagator $P(a,b)$, given by different directions of z in the complex plane, might correspond to different universes. The preceeding notion is developed along similar lines as Weinberg's generalization of the Einstein field equations, where a cosmological constant is introduced, effecting greater generality in the formulation of the equations, and different values of this constant are proposed to correspond to different universes. In a similar fashion, we take each different particle propagator, resulting from different limits of the complex time-slicing index z, as physical laws corresponding to distinct universes.

When this generalization is made, an integral must be computed over a complex number of distinct variables, and as such, a novel calculus was developed to determine the values of such expressions. In essence, the mathematical formalism of this theory follows similar lines as the fractional calculus, by generalizing certain properties of multiple integrals over an integer number of variables, to the case of complex dimension. For instance, the equation

$$\int\int \dots \int \prod_{j=1}^{n} f(x_j)\, dx_1 \dots dx_n = \left(\int f(x)\, dx\right)^n \quad (1.2)$$

concerning the standard separation-of-variables property, is held to remain true even when $n \in \mathbb{N} \rightarrow z \in \mathbb{Z}$. Additional axioms of this calculus are also generalized from the integer case, including the substitution of variables, and the decomposition of functions into a Fourier integral. As a result, expressions of the form (1.1) can be interpreted even in the case of complex index, therefore allowing this generalization of path-integration to be effected.

In addition to generalizing the procedure of integration itself to the case of complex index, we also needed to generalize the path contribution formula, or integrand, to this new domain. As currently expressed, the Feynman contribution formula $exp\left(\frac{i}{\hbar}S\right)$ applies only to paths of integer discretization index, as this is the only domain where path-integration has been developed. Indeed, the formula $exp\left(\frac{i}{\hbar}S\right)$ is not purely motivated by the theoretical framework, and any number of other contribution formulae would be mathematically consistent and yield converging values for (1.1); this expression for the contribution formula is rather fixed by experiment, and this choice of path-integrand is necessary for the propagator expressed in (1.1) to be consistent with experiment, and produce the known form of the nonrelativistic Schrodinger equation. As such, there is no particular reason to expect that the contribution formula or integrand of generalized path-integration, where the index becomes complex, need be of the form $exp\left(\frac{i}{\hbar}S\right)$. In fact, any contribution formula F such that $F \rightarrow exp\left(\frac{i}{\hbar}S\right)$ where the time-slicing index $z \in \mathbb{N}$ is equally consistent in this regard, coinciding with the known formula in the region of integer time-slicing index where all experiments have been made.

Thus, there are many options available for generalizing the path contribution to the domain of complex time-discretization index. As such, we use primarily mathematical motivation to develop a reasonable generalization. We take as fundamental axioms the following postulates:

1. That the value of such expressions as (1.1), for complex n, be determined through generalizations of known properties of integer-dimensioned multiple integrals;
2. That the path-contribution formula is always a *pure phase factor* dependent upon the action.

This latter property itself forms the basis of Feynman's original path-integral framework, where Feynman's two postulates concern 1. That all paths contribute to the propagator, and 2. This contribution is given by a phase, dependent upon the action. It is indeed this phase property that results in the convergence upon the classical limit, as further explicated in Section II. As such, it appears reasonable to generalize these two postulates to complex index. Unfortunately, when the index is so generalized, the action S is no longer real, and the formula $exp\left(\frac{i}{\hbar}S\right)$ no longer a pure phase-factor. To amend this, we generalize the contribution rather as

$$exp(i\,\mathrm{norm}(S)), \quad (1.3)$$

which reduces to the standard integrand for real time-slicing index, where S is real (the case of negative action is considered carefully in [5]).

We chose the p-norm for (1.3), as this general class of norms on $\mathbb{C}$ includes most useful norms on the complex numbers, including the Euclidean, taxicab, and maximum norms. As such, (1.3) reduces to a formula of the form $exp\left(\frac{i}{\hbar}|S|_p\right)$. The action S is most reasonably defined as a (complex-order) sum, over each discretized leg of the path, of the integral of the Lagrangian over this leg. This definition reduces to

$$\sum_{j=0}^{n-1}\int_{t_j}^{t_{j+1}}\frac{m}{2}\left(\frac{(x_{j+1}-x_j)}{T}N\right)^{\mathbf{2}}-\mathrm{V}\,dt \quad (1.4)$$

When such definitions for the path contribution are introduced to (1.1), and the new integration methods are thusly used, a spectrum of possible results are determined for the limit, corresponding to distinct particle propagators. These distinct results depend only upon the phase of the complex time-discretization index, and on the choice of p-norm to complete the generalization of the functional integrand. Although the value of this integrand, as in (1.3), is itself the exponential of a real number, the process of integration over a complex *number of* variables introduces non-trivial complex factors into the result. As [5] demonstrates, in the case of a free particle, this theory effectively introduces a complex factor multiplying the mass, given by

$$\frac{|\exp(it)|_p}{exp(it)} \quad (1.5)$$

which falls within a particular region of the complex plane, shown in Fig. 1.1.

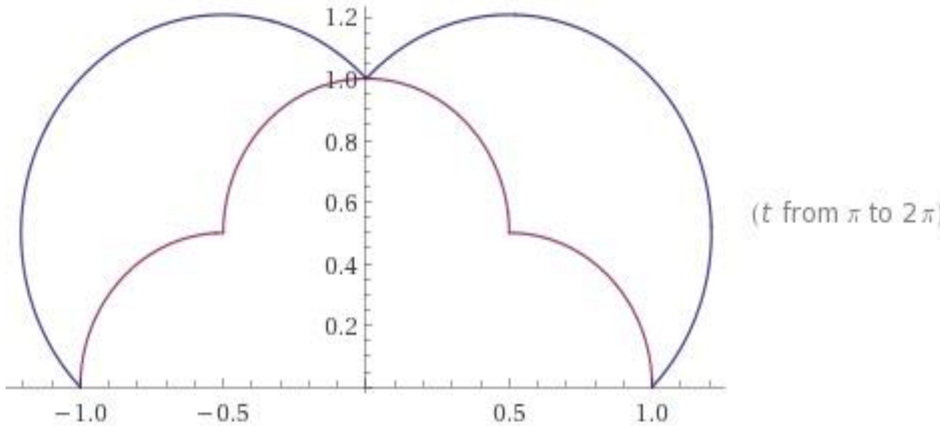

Fig. 1.1: The locations of the complex mass multipier in the complex plane are confined within the figure above.

When a potential is considered, a perturbation expansion must be used to determine the value of the propagator. This process is effected in [5], where a generalized Schrodinger equation is developed. This equation is given by the schema

$$-\frac{\hbar^2}{2m|\exp(i\mu)|_p}\frac{\partial^2\psi}{\partial x^2}+\{|\cos(\mu)|^p-|\sin(\mu)|^p\}\left(|\cos(\mu)|^p+|\sin(\mu)|^p\right)^{\frac{1}{p}-1}V\psi$$

$$=\exp(-\mathrm{i}\mu)\, i\hbar\frac{\partial\psi}{\partial t} \quad (1.6)$$

where the parameter μ, giving the discretization index phase, runs from π to 2π, while the norm parameter p is constrained to be greater than 1. Each different value of p and μ, corresponding to a point within the boundaries of Fig. 1.1, corresponds to a distinct version of the Schrodinger equation (1.6), interpreted as applying to a separate universe. In essence, each universe has a distinct effective mass multiplier (the point in Fig. 1.1), as well as a different factor $\{|\cos(\mu)|^p-|\sin(\mu)|^p\}\left(|\cos(\mu)|^p+|\sin(\mu)|^p\right)^{\frac{1}{p}-1}$ multiplying the potential.

B. Overview of Quantum Temporal Dynamics and the One-Electron Universe

In [4], the authors developed a novel theoretical framework to support Wheeler's notion of a one-electron universe. The latter idea developed from a consideration of the Feyman diagrams of quantum field theory, and particularly how antiparticles behave in the same fashion as particles moving in a reverse temporal direction. Wheeler's idea was that all electrons in the universe are, in fact, the *same* electron, the positron representing this particle reversing directions in time. The relativistic world-line of the electron therefore might appear to be a highly convoluted, "zig-zag" path, and the cross-section of space with this path contains separate electrons wherever this world-line intersects. The one-electron idea has, however, fallen our of favor, not least because it would require equal amounts of antimatter as physical matter. Various mechanisms of circumventing this, for example by postulating the existence of positrons “hiding” within the nucleus, have not met with experimental confirmation. Nonetheless, the one-electron universe model still serves as a useful pedagogical technique for introducing certain aspects of quantum field theory.

[4] developed a generalization of standard, nonrelativistic quantum mechanics, quantum temporal dynamics, that provides a possible mechanism for the one-electron universe. In usual nonrelativistic quantum theory, space is an operator while time is simply a parameter indexing the evolution of quantum states. As such, while wavefunctions and probability densities extend over the three spatial dimensions, they do not extend over time in the same fashion. Namely, one cannot speak of the probability of finding a particle in a certain temporal range, whatever that would mean, like one can speak of finding it in a certain spatial range. As such, while spatial kinematics and motion are consequences of the Schrodinger equation, the “motion” of particles through time goes unexplained.

Quantum temporal dynamics corrects this discrepancy, by describing temporal motion as a consequence of a more generalized Schrodinger equation. In order for QTD to work, two temporal dimensions must be considered, the time internal to a particle or system, and the time external to a particle or system. Internal time indexes the evolution of the particle’s state, while external time indexes the interactions between particles and systems. Two systems can only interact if they coincide in external time. However, there is no such causal restriction on internal time, meaning that a particle can potentially interact with its past or future self.

Indeed, the latter notion forms the crux of QTD’s one-electron universe. For simplicity, we consider a nonrelativistic universe consisting only of a single electron, with only an electrostatic force interaction given by the Coulomb potential, with instantaneous force transmission. Of course, our actual universe is far more complicated than this, but these results might be generalized to the relativistic framework of field theory. Furthermore, each different type of particle in our universe, for example quarks and muons, might correspond analogously to a single particle. The one electron in the one-electron universe of QTD reflects between two potential barriers located in the far future and distant past of external time, and the intersections of this world-line with any cross-section at a particular value of external time appear to us as separate particles.

Thus, the potential at any point of external time, and spatial position, is due entirely to this single particle. As the preceeding argument demonstrates, any internal-time state of the particle could conceivably contribute to this potential, so we should form an integral over all over internal time to determine the value of this potential. In [4], and in Section 2.1 of this paper, this formula is developed, given by

$$V(x,t) = R(t) + \iiint |\psi(\boldsymbol{r}', t'; T')|^2 V(\boldsymbol{r} - \boldsymbol{r}')\delta_\varepsilon\,(t - t')d\boldsymbol{r}'dt'dT' \quad (1.7)$$

where t is external time and T is internal time. Here, the generalized Dirac Delta exponential $\delta_\varepsilon(x) = \frac{1}{\varepsilon\sqrt{2\pi}} exp\left(-\frac{x^2}{2\varepsilon^2}\right)$ determines the role of external time, by promoting contributions to the integral corresponding to states of the source particle close in external time. $R(t)$ describes the vast potential barriers in the future and past of external time. The function $V(\boldsymbol{r} - \boldsymbol{r}')$ is just the spatial Coulomb potential, given as $\frac{Dq^2}{|\boldsymbol{r}-\boldsymbol{r}'|}$ with q the charge on the electron and D some constant, whose value is determined in [4]. In that paper, we suggested that the value of ε, fixing the relative scale of violations of external time causality, could be roughly on the order of the Planck time. The reason we allowed such violations, and did not use a pure Dirac Delta function, was that the temporal wavefunctions are spread out over external time, and as such allowing this "fuzziness" seemed natural.

The second component of quantum temporal dynamics is the generalized Schrodinger equation, delineating the evolution of quantum temporal states in external time. We consider a wavefunction $\psi(\boldsymbol{r}, t; T)$ over space and external time, evolving with internal time. The standard Schrodinger equation is obviously given as

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t} \quad (1.8)$$

Given that the d'Alembertian is simply the Laplacian on four-dimensional space-time, we thought it natural to generalize this equation to the temporal case as

$$-\frac{\hbar^2}{2m}\Box\psi + V\psi = i\hbar\frac{\partial\psi}{\partial T} \quad (1.9)$$

However, (1.9) must be modified to be made consistent with the true nature of the universe. A rescaling factor must be introduced before the temporal derivative, as the succeeding section of this paper and [4] make clear. Thus, we have

$$-\frac{\hbar^2}{2m}\Box_a\psi(\boldsymbol{r}, t; T) + V\psi(\boldsymbol{r}, t; T) = i\hbar\frac{\partial\psi(\boldsymbol{r}, t; T)}{\partial T} \quad (1.10)$$

where the modified d'Alembertian $\Box_a = \nabla^2 - \frac{1}{a^2}\frac{\partial^2}{\partial t^2}$. The constant a is a free parameter of the theory, the "temporal velocity," whose nature is further explained in the succeeding section. We note that the negative sign before the temporal derivative above is reversed in the succeeding part of this paper, when quantum temporal dynamics is developed from the path-integral formulation instead.

Substituting (1.7), the generalized Schrodinger equation (1.10) yields

$$-\frac{\hbar^2}{2m}\Box_a\psi(\boldsymbol{r},t;T)+\left\{R(t)+\iiint|\psi(\boldsymbol{r}',t';T')|^2V(\boldsymbol{r}-\boldsymbol{r}')\delta_\varepsilon(t-t')d\boldsymbol{r}'dt'dT'\right\}\psi(\boldsymbol{r},t;T)$$

$$=i\hbar\frac{\partial\psi(\boldsymbol{r},t;T)}{\partial T}\quad(1.11)$$

Assuming a simplified potential of the form $V(\boldsymbol{r})+R(t)$ that doesn't change over external time, and by introducing a separation of variables,

$$\psi(\boldsymbol{r},t;T)=\theta(\boldsymbol{r},T)\phi(t,T)\quad(1.12)$$

describing the quantum state as a product of spatial and temporal wavefunctions independently, we can decouple (1.11), producing

$$-\frac{\hbar^2}{2m}\nabla^2\theta(\boldsymbol{r},T)+V(\boldsymbol{r})\theta(\boldsymbol{r},T)=i\hbar\frac{\partial}{\partial T}\theta(\boldsymbol{r},T)\quad(1.13)$$

$$\frac{\hbar^2}{2ma^2}\frac{\partial^2}{\partial t^2}\phi(t,T)+R(t)\phi(t,T)=i\hbar\frac{\partial}{\partial T}\phi(t,T)\quad(1.14)$$

The form of (1.14) immediately lends credence to the notion of "temporal duality" developed in [4], namely that the temporal wavefunction ϕ behaves in exactly the same fashion as the standard spatial wavefunction, but rescaled by the temporal velocity coefficient a. As such, motion through external time can be understood in much the same way as motion through space. We imagine that the form of ϕ might be a very narrow wavepacket, reflecting between the great potential barriers defined by $R(t)$. It should also be noted that (1.13) is simply the standard Schrodinger equation, but with internal time in the place of standard time. As such, (1.13) demonstrates that it is internal time that evolves the quantum state of the particle, in accordance with our interpretation listed above.

C. Synthesis of QTD with Multiphase Discretization

The present paper firstly endeavours to develop Quantum Temporal Dynamics from the path-integral formulation, and thus motivating the generalized Schrodinger equation (1.11) from the basic axioms fixing the contribution formulae for paths through external time as well as space. Once such a path-integral formulation of QTD is developed, QTD can be applied in the case of a complex time-discretization index. As such, the temporal behavior of particles in the alternate universes induced by mutliphase discretization can be determined. As Section 5 demonstrates, according to the combination of QTD and Complex Discretization, particles in these parallel universes should travel slower through time than in our own universe, opening possibilities of superluminal travel and communication, described in Section 5.

Furthermore, the cosmology of QTD, developed in Section 4, suggests a possible physical location for the parallel universes of Section 1.1. As we argue below, these universes might exist in our future or past, separated by a quantum potential barrier. The single electron populates these universes by tunneling through these barriers. These chronologically separated domains might offer the location of the parallel dimensions explored previously.

II. Path-Integral Formulation of Quantum Temporal Dynamics

A. Fundamental Postulates of Temporal Dynamics

The framework of quantum temporal dynamics distinguishes between the *internal time* of a system, which indexes the change of its quantum state, and the *external time*, which indexes its relationship with other systems. As such, two systems can only interact if they coincide in external time; thus, a system can interact with its past state, or future state, if they coincide in external time, even if they differ significantly in internal time. In the one-electron universe, with large potential barriers at the past and future, the potential at a particular point of space-time is due entirely to a single particle, whose state evolves according to a nonlinear self-interaction as in (1.11).

In deriving this model of temporal dynamics from the functional-integral approach, given that internal time evolves the system while external time is an observable, it seems natural to treat external time, at least in the context of the free particle, in the same fashion as the spatial dimensions. The difference between external time, and a fourth spatial dimension, comes about in time's role as determining the interaction between systems, as in (1.7).

As such, let us consider the path-integral Lagrangian (1.4), but with multiple spatial dimensions:

$$\sum_{j=0}^{n-1}\int_{t_j}^{t_{j+1}}\frac{m}{2}\left(\frac{\left(x_{j+1}-x_j\right)^2+\left(y_{j+1}-y_j\right)^2+\left(z_{j+1}-z_j\right)^2}{T^2}N^2\right)-\mathrm{V}\,dt \quad (2.1)$$

Introducing the external time parameter analogously, we have

$$\sum_{j=0}^{n-1}\int_{t_j}^{t_{j+1}}\frac{m}{2}\left(\frac{\left(x_{j+1}-x_j\right)^2+\left(y_{j+1}-y_j\right)^2+\left(z_{j+1}-z_j\right)^2+\left(\tau_{j+1}-\tau_j\right)^2}{T^2}N^2\right)-\mathrm{V}\,dt \quad (2.2)$$

where, to prevent confusion, we have relabelled external time as τ and internal time as t (where T is the total duration of internal time). However, as explicated in depth in [4], regarding the analogous Schrodinger formulation, this provisional formula is inconsistent with the known properties of the universe, and must be adequately modified. In essence, we can understand this as a consequence of time being more "rigid," in a sense, than space. That is, deviations from the standard classical trajectory through time are far smaller than the corresponding deviations in spatial motion. While in classical Lagrangian mechanics, the path of lowest action is always traversed, in the quantum path-integral formulation all paths contribute to the propagator; however, due to the phase factor $exp\left(\frac{i}{\hbar}S\right)$, paths with minimal action tend to reinforce their contributions, wheras paths with higher action have cancelling contributions. As S increases from the minimum action, changes in path correspond to smaller and smaller phase periods, thus leading to greater cancellation. Hence, in the classical limit, only the path of minimal action contributes to the propagator in a non-negligible way.

By "rigidity" of time, we mean that deviations of the path with respect to the external time parameter, from the classical path of least action, contribute to the propagator far less than corresponding deviations in the spatial coordinates. As such, let us multiply the external time parameter in (2.2) by some factor, to magnify the effect of changes of the path in external time:

$$\sum_{j=0}^{n-1}\int_{t_j}^{t_{j+1}}\frac{m}{2}\left(\frac{\left(\boldsymbol{r}_{j+1}-\boldsymbol{r}_j\right)^2+a^2\left(\tau_{j+1}-\tau_j\right)^2}{T^2}N^2\right)-\mathrm{V}\,dt \quad (2.3)$$

where henceforth we shall abbreviate $(x_i-x_{i-1})^2+(y_i-y_{i-1})^2+(z_i-z_{i-1})^2$ as $(\boldsymbol{r}_i-\boldsymbol{r}_{i-1})^2$. Here, a is a free parameter of the theory, the "temporal velocity." In [4], we argued that the value of a, to be consistent with experiment, must be roughly 10^{80} c. As Section IV will demonstrate, when combined with the multiphase discretization model, this must be amended to roughly 10^{100} c.

The addition of a might be considered arbitrary, as a substitution of variables could be introduced to subsume it, as in $\tau'_i-\tau'_{i-1}=a(\tau_i-\tau_{i-1})$. However, the nature of τ, as distinguished from any rescaled temporal parameter, is that it determines the same measurements as our standard units of time. That is, particle states evolve relative to t in such a manner that they obey the standard Schrodinger equation relative to this parameter, as in [1.13]. Thus, the specific parameter τ is such that the velocity of the particle through time, as given by the ratio of external time traversed to internal time, is nearly exactly 1. Hence, the two measures of time are consistent as they are traditionally used.

The equation (2.3), the Lagrangian for a path-integral temporal dynamics, is the first major postulate of the theory, expressing how a particle's location in external time evolves with internal time (by considering external time on the same footing as traditional spatial dimensions in the propagator). The second postulate concerns what was expressed at the end of 1.2, namely the nature of external time as defining the relationships between particles or systems. In our one-electron universe, we postulate that the potential at any point in space-time is due entirely to the influence of one particle, whose future and past states correspond to apparently different particles at any value of external time. Thus, all different states of the particle, corresponding to distinct values of internal time, can hypothetically contribute to this potential. Therefore, given the temporal wavefunction developed in (1.11), we have

$$V(\boldsymbol{r},\tau) = \iiint |\psi(\boldsymbol{r}',\tau';t')|^2 f(\boldsymbol{r}-\boldsymbol{r}',\tau-\tau')\, d\boldsymbol{r}' d\tau' dt' \quad (2.4)$$

where we integrate over all states of the particle at different internal times t to determine the value of the potential. Here, the contribution of the differential component of the temporal wavefunction ψ, at some location $(\boldsymbol{r}',\tau';t')$, is determined solely by its relative position in space and external time, given by the function $f(\boldsymbol{r}-\boldsymbol{r}',\tau-\tau')$. Generally, as τ indexes particle interations, significant differences $\tau-\tau' \gg 0$ diminish the value of the function f significantly. Only for $\tau-\tau' \approx 0$ is there a significant contribution to the potential.

In the case of electrostatic potential, which our simplified model assumes, we might ask why the form of f expressed in (1.7) is necessary; namely, why can't f simply adopt the standard form of the electrostatic potential, but with an additional spatial dimension, thus

$$f(\boldsymbol{r}-\boldsymbol{r}',\tau-\tau') = \frac{s}{\sqrt{(\boldsymbol{r}-\boldsymbol{r}')^2 + a^2(\tau-\tau')^2}} \quad (2.5)$$

for some constant s. If a is indeed sufficiently large, then $\tau-\tau' \gg 0$ will correspond to small values of f, consistent with our intended intepretation of external time. Futhermore, $\tau-\tau'=0$ reduces to the standard potential formula, as given in Section 1.B. The difficulty here is that if we take values of internal time t' so that the temporal wavefunction is localized around an external time τ' that very slightly differs from τ, we can constrain $a^2(\tau-\tau')^2$ to have any arbitrary small but finite value. If the temporal wavefunction is not sufficiently thin to be so localized, then this problem also emerges, even when integrating the state at one value of internal time. As $\frac{s}{\sqrt{(\boldsymbol{r}-\boldsymbol{r}')^2+d}}$ for such a value d is not related to the standard potential formula by a simple factor, this choice of f is not consistent with the known properties of electrostatic potential. Thus, we are led rather to consider that form expressed in (1.7); namely,

$$V(\boldsymbol{r},\tau) = \iiint |\psi(\boldsymbol{r}',\tau';t')|^2 V(\boldsymbol{r}-\boldsymbol{r}')\delta_\varepsilon(\tau-\tau')\, d\boldsymbol{r}' d\tau' dt' \quad (2.6)$$

with an exponential Dirac Delta δ_ε of width ε. As explicated in [4], this formula produces the known experimental result for the potential due to a quantum wavefunction, the states of the single particle at different values of internal time t' apparently behaving as distinct particles for any "cross-section" of space-time at some external time τ. It must be again emphasized that this framework is non-relativistic, as the given Lagrangian is not relativistically invariant.

B. The Free Temporal Particle

Let us consider the behavior of our single particle, absent the form of the potential described in (2.6). As such, we apply the path contribution formula, determined by (1.4) and (2.1), to a free particle where the potential V vanishes, meaning that the electrostatic force considered in this model is assumed vanishing. A brief analysis of (1.4) suffices to demonstrate that the notion of temporal duality, explicated in Section 1.2 and [4], is justified from these path-integral axioms. Namely, the introduction of the external time-coordinate is equivalent to the introduction of a second spatial coordinate, but scaled by the "temporal velocity" constant a. Thus, acknowledging the separability of the corresponding Schrodinger equation as described in Section 1.2, we note that the motion of the particle through external time satisfies much the same principles as free quantum motion through a single spatial dimension, but modified by the $1/a$ term.

Alternatively, to derive the propagator directly, we follow the discretization program outlined in [4], rewriting (1.1) as

$$\lim_{n\to\infty} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{n-1} \left\{x_{j+1}-x_j\right\}^2 + a^2\left(\tau_{j+1}-\tau_j\right)^2 \right) dx_1 d\tau_1 \dots dx_{n-1} d\tau_{n-1} \quad (2.7)$$

using the Lagrangian in (1.4), where $A = \frac{i}{\hbar}\frac{m}{2}\frac{n}{T}$. Here, T is the total interval of internal time, while we are simplifying by considering only a single spatial dimension, in addition to external time (these results trivially generalize to multiple spatial dimensions). First, we note that this expression readily separates to integrals individually over space and external time, as

$$\lim_{n\to\infty} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{n-1} \left\{x_{j+1}-x_j\right\}^2 \right) dx_1 \dots dx_{n-1} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{n-1} a^2\left(\tau_{j+1}-\tau_j\right)^2 \right) d\tau_1 \dots d\tau_{n-1} \quad (2.8)$$

Performing a standard variable substitution, we have

$$\lim_{n\to\infty} \iiint \dots \int \quad exp\left(A\left\{\sum_{j=1}^{n-1}\{z_j\}^2 + \left(x_n - x_0 - \sum_{i=1}^{n-1} z_i\right)^2\right\}\right) dz_1 \dots dz_{n-1} \quad \iiint \dots \int \quad exp\left(Aa^2\left\{\sum_{j=1}^{n-1}\{w_j\}^2 + \left(\tau_n - \tau_0 - \sum_{i=1}^{n-1} w_i\right)^2\right\}\right) dw_1 \dots dw_{n-1} \quad (2.9)$$

where $z_j = x_j - x_{j-1}$ and similarly for w_j. For simplicity's sake, we now simplify just the latter integral, the former following trivially. Introducing a Fourier integral expansion, we have

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} exp\left(\frac{k^2}{4a^2A}\right) \iiint \dots \int \quad exp\left(a^2A\sum_{j=1}^{n-1}\{w_j\}^2\right) exp\left(ik\left(\tau_n - \tau_0 - \sum_{i=1}^{n-1} w_i\right)\right) dz_1 \dots dz_{n-1}\, dk \quad (2.10)$$

Applying the variable separation of (1.2), we have, simplifying this integral,

$$\lim_{n\to\infty} \int_{-\infty}^{\infty} exp\left(\frac{k^2}{4a^2A}\right) exp\big(ik(\tau_n - \tau_0)\big) \left\{\int \quad exp(a^2Az^2 - ikz)dz\right\}^{n-1} dk \quad (2.11)$$

yielding

$$exp\left(\frac{ma^2\boldsymbol{\tau}^2 i}{2\hbar T}\right) \quad (2.12)$$

where $\boldsymbol{\tau}$ is the total duration of external time traversed by the particle. Indeed, in accordance with the notion of temporal duality, we see that this formula is equal to the propagator for the standard free particle in quantum mechanics, but with the spatial component rescaled by the temporal velocity factor. Applying the same procedure to the spatial integral, we have

$$exp\left(\frac{m\mathrm{X}^2 i}{2\hbar T}\right) exp\left(\frac{ma^2\boldsymbol{\tau}^2 i}{2\hbar T}\right) \quad (2.13)$$

which is again consistent with our framework of temporal duality, and with the generalized Schrodinger equation expressed in (1.11).

C. Deriving the Schrodinger-Equation for the One-Electron Universe

Applying the path-integral framework to the particle in a potential is a relatively straightforward application of the perturbative method, as expressed in [5]. In quantum temporal dynamics, where the universe is modelled as a one-electron system, the potential at any point in space-time is due entirely to this single particle, given as an integral of contributions over the particle's entire internal time history, as expressed in (2.6). Incorporating the large potential barriers at the being and end of the universe, that induce the reflections of the single particle, as described in Section 1, we have as our potential

$$V(\boldsymbol{r},\tau) = R(\tau) + \iiint |\psi(\boldsymbol{r}',\tau';t')|^2 V(\boldsymbol{r}-\boldsymbol{r}')\delta_\varepsilon(\tau-\tau')\, d\boldsymbol{r}' d\tau' dt' \quad (2.14)$$

where $R(\tau)$ constitutes these large potential barriers. Assuming the function ψ is given, this expression defines a function of $(\boldsymbol{r},\tau)$, the spatial and external time coordinate. In the simplification of a single spatial dimension x, we have

$$V(x,\tau) = R(\tau) + \iiint |\psi(x',\tau';t')|^2 V(x-x')\delta_\varepsilon(\tau-\tau')\,dx'd\tau'dt' \quad (2.15)$$

where, if ψ is known, $V(x,\tau)$ is again a well-defined function of space and external time. Using the Lagrangian defined in (2.3), we have that the functional integrand, the contribution corresponding to each path, is given as

$$exp\left(\frac{i}{\hbar}\left(\left[\sum_{j=0}^{n-1}\frac{mn}{2}\frac{\{x_{j+1}-x_j\}^2 + a^2(\tau_{j+1}-\tau_j)^2}{T}\right]\right.\right.$$
$$\left.\left.-\frac{T}{n}\sum_{j=0}^{n-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right)\right) \quad (2.16)$$

Incorporating this contribution formula into the functional integral, the expression in (1.1), we have

$$\lim_{n\to\infty}\left(K - \frac{i}{\hbar}\frac{1}{n}\int\int\ ...\int\ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left(T\sum_{j=0}^{n-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j\right.\right.$$
$$\left.\left.+(\tau_{j+1}-\tau_j)s)ds\right)dx_1 d\tau_1 \dots dx_{n-1}d\tau_{n-1} + \cdots\right) \quad (2.17)$$

where $\mathrm{S}[0]$ is the free-particle action given by (2.16) with $V=0$, and K is the free-particle propagator given in (2.13). Now, when deriving the Schrodinger equation, we will assume an infinitesimal internal time-translation T, as Feynman does in [2] and we perform in [5]. This simplification allows us to transform (2.17) into

$$\lim_{n\to\infty} K - \frac{i}{\hbar}\frac{1}{n}\int\int\ ...\int\ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left[exp\left(T\sum_{j=0}^{n-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right)\right.$$
$$\left.-1\right]dx_1 d\tau_1 \dots dx_{n-1}d\tau_{n-1} + \cdots \quad (2.18)$$

where the potential term is written in exponential form, assuming again the infinitesimal nature of the internal time interval. Now, as we are assuming the total internal time-interval to be infinitesimal, we can further stipulate that $x_j + (x_{j+1}-x_j)s \to x_a$, as the argument of the potential in the integral is now dominated by the path's initial starting point rather than the relatively infinitesimal displacement, and likewise for external time, allowing us to write

$$\lim_{n\to\infty} K - \frac{i}{\hbar}\frac{1}{n}\int \int \ . \ . \ . \int \ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left[exp\left(T\sum_{j=0}^{n-1} V(x_a, \tau_a)\right) - 1\right] dx_1 \ ... dx_{n-1} \ + \cdots \ (2.19)$$

The straightfoward substitution and separation of variables, and Fourier integral expansion, as applied in the previous sub-section, but now with the appearance of the sum $\sum_{j=0}^{n-1} T\, V(x_a,, \tau_a)$, yields

$$\lim_{n\to\infty} K - \frac{i}{\hbar}\frac{1}{n}\left(Kexp\big(nTV(x_a, \tau_a)\big) - K\right) + \cdots \ (2.20)$$

Which, for small T, gives

$$\lim_{n\to\infty} K - \frac{i}{\hbar}\big(KTV(x_a, \tau_a)\big) + \cdots \quad (2.21)$$

Producing[1]

$$K - \frac{i}{\hbar}TV(x_a, \tau_a)K + \cdots \ (2.22)$$

Applying the definition of the wavefunction in terms of the propagator, as we do in [5] and Section 3.3 below, results in the equation

$$-\frac{\hbar^2}{2m}\Box_a\psi(x, \tau; t) + V(x, \tau)\psi(\boldsymbol{r}, \tau; t) = i\hbar\frac{\partial\psi(x, \tau; t)}{\partial t} \quad (2.23)$$

where $\Box_a = \nabla^2 + \frac{1}{a^2}\frac{\partial^2}{\partial t^2}$. In the single spatial dimension we are considering, the Laplacian ∇^2 readily reduces to the partial derivative $\frac{\partial^2\psi(x,\tau;t)}{\partial x^2}$. This equation is simply the standard Schrodinger equation in two spatial dimensions, but with the second dimension (delineating external time) rescaled, and the potential $V(x, \tau)$ given by the formula (2.15). Application of separation of variables into wavefunctions over space and external time separately readily results in (1.13-14), which defines the duality of temporal and spatial motion.

D. Differences with the Quantum Field Theory Promotion of Time to an Operator

The notion of promoting time to an operator, rather than demoting space to a parameter, has been explored before in the context of quantum field theory, for example in [6]. As such, a Schrodinger equation results, defining the dynamics of the particle's motion through time. In order to work, this framework requires two separate temporal dimensions, the operator considered above, and a temporal paramater to evolve quantum states. The latter is the relativistic proper

[1] [5] provides a full derivation of the analogous formula for complex time-discretization index.

time of the particle, while the former is the time as measured in some reference-frame external to the particle.

We note that our theory, which also describes two temporal components (internal and external to the particle), one of which is promoted to operator, is unique and not in any way similar to the quantum field theory notion. Firstly, quantum temporal dynamics is a nonrelativistic theory, and therefore separate from the mechanisms of quantum field theory. The motivation to promote time to operator is not to ensure relativistic invariance or space-time symmetry; indeed, the form of the Lagrangian in (2.3) is not relativistically invariant. As such, there is no room for the relativistic notion of “proper time” in this theory. The difference between internal and external time is completely separate from the relationship between proper time, and time as measured by some external observer. The latter two notions are linked by relativistic kinematics, meaning that if the particle is moving quickly relative to that observer, then its speed through external time, measured relative to its proper time, will greatly increase, while there is no such relationship in quantum temporal dynamics. The speed of a particle in QTD through external time, as measured by the ratio of external time traversed to internal time elapsed, bears no relationship to the spatial velocity of the particle in some reference-frame, as the temporal duality expressed in [1.14] makes perfectly obvious.

Furthermore, the notion of promoting time to operator has never before been linked to the one-electron universe hypothesis. As the previous Section describes, the notion of quantum temporal dynamics was developed specifically to support a dynamical version of the one-electron universe. As such, the potential at any point in space-time is given solely by the single particle, by an integral over its entire future and past history. External time coordinates serve, through a generalized Dirac Delta function, to determine the relative contribution of each internal time state of the particle to this potential, while large potential barriers are present at the beginning and end of the universe. These notions are entirely absent from the QFT formulations involving proper time as parameter and some external time coordinate as operator. Namely, the self-interaction formula (2.15) and Lagrangian (2.3), yielding the temporal Schrodinger equation (2.23), do not appear in any previous work.

III. Quantum Temporal Dynamics with Complex Discretization Index

A. Solving the Free-Particle of Temporal Dynamics for Complex Discretization

The purpose of the present paper is to combine the mathematical formalisms of quantum temporal dynamics with that of path-integration for complex discretization index. As such, we shall apply the results of the previous section to the case of complex slicing index, and derived a generalized form of the Schrodinger equation that specifies the temporal dynamics given a discretization index of complex phase. In essence, this shall be a temporal version of (1.6) specifying the dynamics of particles in external time, in the different universes specified by alternate phases of the index. First, we follow the general outline of Section 2.2, and derive the behavior of the free temporal particle, without the presence of the self-interacting potential (2.15).

Firstly, we have the contribution formula (1.3), given formula (2.3) for the Lagrangian, reduce to

$$exp\left(\frac{i}{\hbar}\left|\sum_{j=0}^{z-1}\int_{t_j}^{t_{j+1}}\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)\frac{z^2}{T^2}\,dt\right|_p\right) \quad (3.1)$$

In [5], in developing the mathematical mechanism of such functional integration, we assume that the real or imaginary part of a sum is the sum of the real or imaginary parts of the terms,

$$Re\left(\sum_{j=0}^{z-1}\int_{t_j}^{t_{j+1}}\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)z^2\,dt\right)$$

$$=Re(z)\left[\sum_{j=0}^{z-1}T\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)\right] \quad (3.2)$$

Likewise,

$$Im\left(\sum_{j=0}^{z-1}\int_{t_j}^{t_{j+1}}\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)z^2\,dt\right)$$

$$=Im(z)\left[\sum_{j=0}^{z-1}T\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)\right] \quad (3.3)$$

Implying that

$$\left|\sum_{j=0}^{z-1}\int_{t_j}^{t_{j+1}}\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)z^2\,dt\right|_p$$

$$=|n|_p\left[\sum_{j=0}^{z-1}T\left(\left\{x_{j+1}-x_j\right\}^2+a^2\left(\tau_{j+1}-\tau_j\right)^2\right)\right] \quad (3.4)$$

Hence, the functional integral (1.1) readily reduces to

$$\lim_{z\to\infty} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{z-1} \{x_{j+1} - x_j\}^2 + a^2 (\tau_{j+1} - \tau_j)^2 \right) dx_1 d\tau_1 \dots dx_{z-1} d\tau_{z-1} \quad (3.5)$$

where

$$A = \frac{i}{\hbar} \frac{m}{2} \frac{|n|_p}{T} \quad (3.6)$$

Now, to compute this expression, we must introduce an additional mathematical postulate to the mechanism expressed in [5], namely that an alternating multiple integral of complex dimension, of the form

$$\iiint \dots \int \quad f\left(\sum_{j=0}^{z-1} g(x_{j+1} - x_j) \right) w\left(\sum_{j=0}^{z-1} h(\tau_{j+1} - \tau_j) \right) dx_1 d\tau_1 \dots dx_{z-1} d\tau_{z-1} \quad (3.7)$$

for analytic and explicitly defined functions f, g, w, h, can be readily decomposed as

$$\iiint \dots \int \quad f\left(\sum_{j=0}^{z-1} g(x_{j+1} - x_j) \right) w\left(\sum_{j=0}^{z-1} h(\tau_{j+1} - \tau_j) \right) dx_1 d\tau_1 \dots dx_{z-1} d\tau_{z-1}$$

$$= \iiint \dots \int \quad f\left(\sum_{j=0}^{z-1} g(x_{j+1} - x_j) \right) x_1 \dots dx_{z-1} \iiint \dots \int \quad w\left(\sum_{j=0}^{z-1} h(\tau_{j+1} - \tau_j) \right) d\tau_1 \dots d\tau_{z-1} \quad (3.8)$$

As such, (3.8) being a postulate for complex-dimensioned integration could be viewed as a similar separation axiom to (1.2), but performing a double, rather than complex-order, separation. Applying (3.8) to (3.5), we have

$$\lim_{z\to\infty} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{z-1} \{x_{j+1} - x_j\}^2 \right) dx_1 \dots dx_{z-1} \iiint \dots \int \quad exp\left(A \sum_{j=0}^{z-1} a^2 (\tau_{j+1} - \tau_j)^2 \right) d\tau_1 \dots d\tau_{z-1} \quad (3.9)$$

The derivation from this point is straightforward, following the mathematical route proscribed in Section 2.2, and in the main body of [5]. Each factor simplifies in the same manner as the standard free-particle for complex discretization index, described in [5], by an application of substitution of variables to separate the exponential factors within the integrand, the use of a Fourier integral expansion to achieve this, and use of (1.2). Thus, the result is simply the multiplication of the two standard propagators produced by each factor of (3.9), namely

$$exp\left(\frac{mX^2 i \left(|\exp(i\mu)|_p \right)}{2\hbar T exp(i\mu)} \right) exp\left(\frac{ma^2 \tau^2 i \left(|\exp(i\mu)|_p \right)}{2\hbar T exp(i\mu)} \right) \quad (3.10)$$

μ being the complex phase of the time-discretization index, running from π to $2\,\pi$, p again determining the choice of norm, and a being the temporal velocity. (3.10) affirms the notion of temporal duality appearing in Section 2.2, explicated in Section 1.2 and [4]. As such, the quantum propagator corresponding to the motion through external time is merely a rescaled form of that corresponding to spatial motion, as the form of the Lagrangian (2.3) makes evident.

B. The Particle in a Potential with Complex Discretization

The determination of quantum temporal dynamics, for complex time-discretization index, in a potential follows straightforwardly from the programme adapted in Section 2.3. Firstly, however, we must note that in the other universes developed in [5], standard unitarity does not apply to the quantum wavefunction, meaning that the wavefunctions solving (1.6) are generally not normalizable. While the authors of [5] did develop a probabilistic interpretation to account for such non-normalizability, unfortunately this interpretation is inconsistent with the cosmological model set forth in Section 4. As such, we shall define the meaning of the wavefunction such that

$$p\,(x',\tau';t') = A(t)|\psi(x',\tau';t')|^2 \quad (3.11)$$

modifying the standard formula for the probability density p with the appropriate normalization factor $A(t)$ over internal time. The wavefunction itself is still given by the standard Feynman integral of the initial wavefunction and particle propagator, as expounded in [2]. As such, for complex discretization index, it appears that the appropriate definition for the potential at a point modifies (2.15) as

$$V(x,\tau) = R(\tau) + \iiint \frac{|\psi(x',\tau';t')|^2}{\int_{x'-\tau'}|\psi(x',\tau';t')|^2 dx'd\tau'} V(x-x')\delta_\varepsilon(\tau-\tau')\,dx'd\tau'dt' \quad (3.12)$$

Again, if the wavefunction ψ is a known function, (3.12) defines a perfectly well-defined function on (x,τ). As such, our strategy shall be to "assume" the wavefunction known, and thus (3.12) a defined function on space-time, carry (3.12) along with the derivation of the generalized Schrodinger equation, and ultimately define a nonlinear mixed integral-differential equation for the wavefunction.

Following the general procedure used in [5], we set the value of the path contribution as

$$exp\left(\frac{i}{\hbar}\left|\int_0^T L\,dt\right|_p\right) =$$

$$exp\left(\frac{i}{\hbar}\left(\left|Re(z)\left[\sum_{j=0}^{z-1}\frac{m}{2}\frac{\{x_{j+1}-x_j\}^2 + a^2(\tau_{j+1}-\tau_j)^2}{T}\right] - TRe\left[\frac{1}{z}\right]\sum_{j=0}^{z-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right|^p\right.\right.$$

$$+\left|Im(z)\left[\sum_{j=0}^{z-1}\frac{m}{2}\frac{\{x_{j+1}-x_j\}^2 + a^2(\tau_{j+1}-\tau_j)^2}{T}\right]\right.$$

$$\left.\left.\left.-TIm\left[\frac{1}{z}\right]\sum_{j=0}^{z-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right|^p\right)^{1/p}\right) \quad (3.13)$$

Inserting this formula into (1.1) in the case of non-zero potential V, expanding the contribution formula in a Taylor series in $-\sum_{j=0}^{n-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j))ds$, and simplifying, we obtain

$$\lim_{n\to\infty}\left(K-\frac{i}{\hbar}B\int\int\ ...\int\ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left(T\sum_{j=0}^{n-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right)dx_1d\tau_1\ldots dx_{z-1}d\tau_{z-1}+\cdots\right)\quad(3.14)$$

In which the ellipsis denotes the sum of the other terms, S[0] is the free particle action, and B is

$$B=\left(Re(n)Re\left(\frac{1}{n}\right)|Re(n)|^{p-2}+Im(n)Im\left(\frac{1}{n}\right)|Im(n)|^{p-2}\right)(|Re(n)|^p+|Im(n)|^p)^{\frac{1}{p}-1}\quad(3.15)$$

Again, to derive an analogue of the Schrodinger equation, as in 2.3, we need only consider infinitesimal time-translations T. For the limit of small T, as in 2.3, we have

$$\lim_{n\to\infty}K-\frac{i}{\hbar}B\int\int\ ...\int\ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left[exp\left(T\sum_{j=0}^{z-1}\int_0^1 V(x_j+(x_{j+1}-x_j)s,\tau_j+(\tau_{j+1}-\tau_j)s)ds\right)-1\right]dx_1d\tau_1\ldots dx_{z-1}d\tau_{z-1}+\cdots\quad(3.16)$$

where the substitution of the exponential is accurate for small T. Furthermore, for such small time-translations, we may make the further simplification, as used in Section 2.3 and [5],

$$\lim_{n\to\infty}K-\frac{i}{\hbar}B\int\int\ ...\int\ exp\left(\frac{i}{\hbar}\mathrm{S}[0]\right)\left[exp\left(T\sum_{j=0}^{z-1}V(x_a,\tau_a)\right)-1\right]dx_1d\tau_1\ldots dx_{z-1}d\tau_{z-1}+\cdots\quad(3.17)$$

as the value of the potential V is nearly exactly $V(x_a,\tau_a)$, over the entire path, for small T. Following the general procedure of Section 2, again with the appearance of the sum $\sum_{j=0}^{n-1}T\,V(x_a)$, we have

$$\lim_{n\to\infty}K-\frac{i}{\hbar}B(Kexp(nTV)-K)+\cdots\quad(3.18)$$

Which, for small T, yields

$$\lim_{n\to\infty}K-\frac{i}{\hbar}Bn(KTV)+\cdots\quad(3.19)$$

Producing

$$K-\frac{i}{\hbar}DTVK+\cdots(3.20)$$

where V denotes the nearly uniform value of the potential along the infinitesimal path (equal to $V(x_a, \tau_a)$ above). In this case,

$$D = \exp(\mathrm{it})\,\{|\cos(t)|^p - |\sin(t)|^p\}\,(|\cos(t)|^p + |\sin(t)|^p)^{\frac{1}{p}-1} \quad (3.21)$$

Given the formula (3.10). Indeed, this is simply the standard formula for complex discretization index obtained in [5], but with the new free-particle propagator and potential, once again lending credence to the general principle of temporal duality. This is straightforward, as this demonstration was little more than an application of complex time-discretization to two spatial dimensions, one of them rescaled, and with the particular potential formula given by (3.12).

C. *The Generalized Temporal Schrodinger Equation*

Using the previously explicated definition of the wavefunction's evolution in terms of the particle propagator, and keeping in mind that the actual probability density is defined by a normalized function as in (3.11), we can now determine the generalized Shrodinger equation. We follow the same programme delineated in [5].

Relevant to the construction of the differential equation of the wavefunction is the behavior of the wavefunction over infinitesimal time-translations, and therefore the propagator over small T. Thus, let us consider the case of an arbitrarily small time-interval $T = \xi$. In this case, the approximations of 3.2 are accurate, and the propagator is given by (10),

$$K - \frac{i}{\hbar} D\xi V K + O(\xi^2) \quad (3.22)$$

or,

$$K exp\left(-\frac{i}{\hbar} D\xi V(x_a, \tau_a)\right) \quad (3.23)$$

Use of variable substitution, power series expansion, and the definition of the wavefunction in terms of the Kernel, as applied similarly in [5], suffices to demonstrate that

$$-\frac{\hbar^2}{2mz}\Box_a \psi + DV\psi = i\hbar \frac{\partial \psi}{\partial t} \quad (3.24)$$

with z given as in (1.5) and D as in (3.21).

Explicitly, the generalized Schrodinger equation is given as

$$-\frac{\hbar^2}{2m|\exp(i\mu)|_p}\Box_a\psi(x,\tau;t)+\{|\cos(\mu)|^p$$

$$-|\sin(\mu)|^p\}\left(|\cos(\mu)|^p+|\sin(\mu)|^p\right)^{\frac{1}{p}-1}\Bigg(R(\tau)$$

$$+\iiint\frac{|\psi(x',\tau';t')|^2}{\int_{x'-\tau'}|\psi(x',\tau';t')|^2dx'd\tau'}V(x-x')\delta_\varepsilon(\tau-\tau')\,dx'd\tau'dt'\Bigg)\psi(x,\tau;t)$$

$$=\exp(-\mathrm{i}\mu)\,i\hbar\frac{\partial\psi(x,\tau;t)}{\partial t}\quad(3.25)$$

with the parameter μ running from π to $2\,\pi$. (3.25) is the central equation describing quantum temporal dynamics in a complex time-discretized universe. As such, it reflects the temporal behavior of particles within the alternate realities described in Section 1.1. The appearance of temporal duality is apparent, in the sense that the equation is readily decomposed into two separate equations, assuming certain approximations of the potential, describing spatial and temporal motion respectively, that are rescaled versions of each other. (3.25) can be so separated using the techniques applied in [4]. At this point, it is interesting to consider how the quantum temporal one-electron cosmology relates to the complex time-discretized multiverse.

IV. Cosmological Multiverse Interpretation

A. Introduction and Formula for the Temporal Velocity (Temporal Stability Coefficient)

In [4], we presented a multiverse cosmology arising from the framework of temporal quantum dynamics. Namely, given the temporal duality expressed in (1.14), the interactions of the model's single electron with the potential barriers $R(\tau)$, from (2.14), should be equivalent to the interactions of a standard particle in one spatial dimension with such a barrier. As such, each reflection from the barrier also carries a small but nonzero chance of quantum tunneling beyond the barrier. If the particle does indeed tunnel in this fashion, beyond the range of external time constituting our "universe," then it will have entered an alternate domain, that we could thusly fashion as constituting an alternate "universe." Indeed, there might be an infinite chain or sequence of such potential barriers, the space between any two barriers constituting a separate universe. When the single electron tunnels through a barrier, it populates the next universe.

This cosmological model affords an interesting opportunity to introduce the multiverse of multiphase discretization. Namely, there is no particular reason to expect that the laws of physics applying in these alternate domains, in our far past and future and separated by vast energy barriers, are the same as in our universe. Indeed, the physical laws applying in these alternate realities could well be very different, as long as they are consistent with the general path-integral framework that forms the basis of quantum temporal dynamics. As such, there is no particular reason to expect that the complex phase of the path-discretization index, which is 0 in our universe, might not adopt a number of different values in these alternate domains. In fact, the multiverse naturally produced by quantum temporal dynamics offers an interesting model for the physical instantiation of the parallel universes of MDM, as well as a physical mechanism (quantum tunneling through the temporal energy barrier) by which these universes are populated. Thus, let us consider such a multiverse.

Firstly, we shall use a physical argument to determine the approximate value of the temporal velocity (or stability coefficient) a. Noting the nature of temporal duality, the temporal wavefunction (as in (1.14)) of the electron should behave in an analogous fashion to the standard quantum wavefunction of a particle in a single spatial dimension. We imagine that such temporal motion is defined by a very tight "wave packet," with the extent of the wavefunction in external time confined roughly to the Planck time. Taking the standard formula for the wavepacket of a particle, and multiplying the mass by the appropriate coefficient to effect the duality, we have (in atomic units)

$$\phi(\tau,t) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{exp\left(\dfrac{-c\tau^2 + i(l\tau - l^2 t/2ma^2)}{1 + 2ict/ma^2}\right)}{\sqrt{1 + 2ict/ma^2}} \quad (4.1)$$

where l is defined as the initial momentum of this wavepacket formation (in some sense, a "temporal momentum"), such that

$$\frac{l}{ma^2} = v \; (4.2)$$

v being the velocity of the particle through external time, while c is a free parameter determining its width in external time. This solution, of course, applies only for real path-discretization, which we shall be using in this sub-section. The use of complex path-discretization, which we shall be considering in the succeeding sub-section and in Section 5, does not significantly alter these results.

Let us suppose that the total duration of internal time traversed by the particle, as it populates the entire multiverse, is T. In the succeeding sub-section we shall develop an approximate value for T. For now, we note that, as in [4], (4.1) gives the probability density (in atomic units)

$$|\phi(\tau,t)|^2 = \sqrt{2/\pi}\, w\, exp(-2w^2(\tau - vt)^2) \quad (4.3)$$

where

$$w = \left(\frac{c}{1 + (2ct/ma^2)^2}\right)^{1/2} \quad (4.4)$$

Applying the same procedure as in [4], we note that the width of the probability density Gaussian is proportional to

$$\frac{1}{w}$$

Thus, the width after an internal time T is given by

$$\left(\frac{1 + (2(1/d)^2 T/ma^2)^2}{(1/d)^2}\right)^{1/2} \quad (4.5)$$

where d is the initial width at internal time $T = 0$. To ensure consistency, the temporal wavepacket should preserve its Planck-scale width over its entire journey. If there are N particles within a particular universe in the QTD multiverse, we can infer that the electron makes N laps between the two potential reflectors $R(t)$ defining the extent of that universe in external time. Assuming that there are M separate universes, and that each universe lasts for an external time Y, the total internal time experienced by our single particle should be on the order of NMY. Thus, the dispersion of the temporal wavepacket over this journey is proportional to

$$\frac{NMY}{mda^2} \quad (4.6)$$

with d the initial width, and the internal time NMY given in atomic units. As the Planck time is 10^{-26} in atomic units, and the electron's mass is 1, our requirement reduces to

$$\frac{NMY}{(a^2 10^{-26})} \approx 10^{-26} \quad (4.7)$$

giving the "effective temporal mass"

$$a^2 = NMY10^{52} \quad (4.8)$$

in atomic units.

B. The Anthropic Principle and Universe Number

Consideration of the anthropic principle, as presented in [5], allows us to determine the approximate number of universes N in the combined MDM-QTD multiverse. Firstly, we must make an assumption concerning the distribution of physical parameters and properties to these distinct universes. Let us therefore assume that the quantum phase parameter μ in (3.25) is distributed evenly among the universes considered in the previous sub-section. If we discover an anthropic constraint concerning the value of μ in our universe, such that it lies in a range of length 1/L, we can therefore estimate that the number of universes is at least on the order of L to accommodate this anthropic constraint without violation of reasonable probabilistic considerations.

Our anthropic constraint derives from a particular property of multiphase discretization, namely that, even in the presence of time-dependent normalization as in (3.11), the law of energy and momentum conservation is violated to varying degrees, depending on the quantum discretization phase μ corresponding to the relevant universe. Let us consider the behavior of quarks within nucelons, and stipulate that the structure of nucleons should be stable, even over long periods of time, within our universe. We can estimate that the time Y, between the hypothetical singularities at the beginning and end of our universe, is on the order of roughly 10^{10} years. Using a nonrelativistic analysis of the quark, we know that, confined within such a small location as a nucleon, by the Heisenberg uncertainty principle it has a proportionally large uncertainty in momentum. Its wavefunction in momentum space should therefore be of the form

$$exp(-p^2/a) \quad (4.9)$$

with a on the order of the quark's momentum uncertainty. Thus, we have the equivalent equation

$$exp(-E/r) \quad (4.10)$$

for the momentum-space equation, where E is kinetic energy and r is the "average" energy from the quantum uncertainty in momentum. It is readily determined from the uncertainty principle that, given the scale of the nucleon's radius, the value of r is roughly 10^9 hartrees. From the spatial Schrodinger equation of the form (1.13) arising from the general equation (3.25), we have that the time evolution of (4.10) is given as

$$exp(-E/r)\, exp(sin(\mu)Et) \quad (4.11)$$

in atomic units, with $t \approx \tau$ the particle's internal time, roughly equal to external time when the velocity v is roughly equal to 1. Thus, if we wish the average energy of the quarks to maintain the same order of magnitude, meaning that their uncertainty in momentum and hence in position also maintain the same order of magnitude, we should have

$$\mu t \approx \frac{1}{r} \quad (4.12)$$

in atomic units, giving us that μ is roughly 10^{-43}. We do not perform this calculation for the whole of the particle's internal-time, just for one "leg" between the temporal barriers, as the nucleons disintegrate and reform at the extremely high-energy environment of the singularities at the beginning and end of the universe. Thus, by the preceeding argument, we have that the number of universes N is on the order of 10^{43}. Hence, we have

$$a^2 = NMY10^{52} = 10^{209} \quad (4.13)$$

given that the number of particles within each universe is roughly 10^{80}, as cosmological observations for our universe suggest, and that each universe lasts for roughly 10^{10} years, on the order of current cyclical cosmological models, giving us the temporal velocity

$$a = 3 \text{ x } 10^{104} \quad (4.14)$$

C. Calculation of Temporal Barrier Strength

Given (4.13), we can now determine the strength of the temporal barriers separating alternate realities. Specifically, we have that the chance of the particle tunneling through such a temporal barrier, as in [4], is given by

$$exp\left(-2ma^2L\sqrt{\frac{U}{E}-1}\right) \quad (4.15)$$

where we assume that the temporal barriers, while tall are extremely thin, with a width in external time given by

$$L = \frac{\hbar}{|E|} \quad (4.16)$$

Thus, (4.15) reduces to

$$exp\left(-4\sqrt{\frac{U}{E}-1}\right) \quad (4.17)$$

Now, we can consider the movements of the electron through the multiverse as analogous to a "random walk" on the real number line. A "bloc" is a period of internal time during which the electron remains confined to a single universe (between two specific barriers). The electron has an equal probability, assuming equal barrier strength, to tunnel to the external-time future or past, and as such the next bloc has an equal probability of being confined to the adjacent future universe, or past universe, of the previous bloc. Therefore, the relevant random-walk to this situation is that on the integers, with probability $p = 0.5$ of motion left or right. After N steps in this model, the farthest distance traversed from the starting point is proportional to $N^{1/2}$. Thus, if we desire the particle to populate 10^{43} universes, it must traverse 10^{86} blocs. There are therefore 10^{43} blocs per universe, meaning that each bloc, as per the above arguments, should include 10^{37} "laps" between the temporal barriers. Thus, we have

$$exp\left(-4\sqrt{\frac{U}{E}-1}\right) = 10^{-37} \quad (4.18)$$

meaning that

$$U = 2.3 \text{ x } 10^{211} \quad (4.19)$$

V. Universe-Dependent Temporal Flow

We have from (4.1) that the motion of the temporal wavepacket through external time, absent an imposed potential, is given by (in atomic units)

$$\phi(\tau,t) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{exp\left(\frac{-c\tau^2 + i(l\tau - l^2 t/2ma^2)}{1 + 2ict/ma^2}\right)}{\sqrt{1 + 2ict/ma^2}} \quad (5.1)$$

Using the generalized Schrodinger equation (3.25), in conjunction with the corresponding temporal duality expressed in (1.14) (from the analogous separation process on the generalized Schrodinger equation), we have the temporal wavepacket

$$\phi(\tau,t) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{exp\left(\frac{-c\tau^2 + i(l\tau - l^2 t/2mza^2)}{1 + 2ict/mza^2}\right)}{\sqrt{1 + 2ict/mza^2}} \quad (5.2)$$

where

$$z = \frac{|\exp(i\mu)|_p}{exp(i\mu)} \quad (5.3)$$

with μ the parameter indexing different universes of the MDM multiverse, lying, as before, between π to $2\,\pi$. Let us consider the speed of this wavepacket through external time. As before, we constrain the temporal momentum l of the above wavefunction such that the speed of the particle through external time, in our universe, is exactly 1. However, when complex path-discretization is invoked, as shown below, the temporal speed of the particle (not the temporal velocity, a constant) varies from universe to universe, the same initial temporal wavepacket (5.2 for $t = 0$) evolving differently for distinct versions of the general Schrodinger equation (3.25).

To ascertain the speed of the temporal wavepacket, we determine the maximum value of its probability density. This density is clearly proportional to

$$\left|exp\left(\frac{-c\tau^2 + i(l\tau - l^2 t/2mza^2)}{1 + 2ict/mza^2}\right)\right|^2 = exp\left(2Re\left(\frac{-c\tau^2 + i(l\tau - l^2 t/2mza^2)}{1 + 2ict/mza^2}\right)\right) \quad (5.4)$$

where the proportionality is given by the normalization function $A(t)$, dependent on the value of internal time. However, this factor $A(t)$, although adjusting the overall time-dependent proportionality of the wavefunction and the potential (3.12), will clearly not affect the position of the maximum of (5.2). This last, by (5.4), is given by the maximum of

$$Re\left(\frac{-c\tau^2 + i(l\tau - l^2 t/2mza^2)}{1 + 2ict/mza^2}\right) \quad (5.5)$$

This reduces to

$$-\frac{l^2t^2csin^2(u)}{m^2a^4b^2\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)}-\frac{l^2t^2ccos^2(u)}{m^2a^4b^2\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)}$$

$$-\frac{l^2tsin(u)}{2ma^2b\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)}$$

$$+\frac{2ltc\tau\cos(u)}{ma^2b\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)}-\frac{w\tau^2}{\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)}$$

$$-\frac{2tc^2\tau^2\sin(u)}{ma^2b\left(\frac{4t^2c^2cos^2(u)}{m^2a^4b^2}+\left(\frac{2tcsin(u)}{ma^2b}+1\right)^2\right)} \quad (5.6)$$

where

$$z = b\,exp(ui) \quad (5.7)$$

The maximum of this expression in external time is given by

$$\tau = \frac{ltcos(u)}{2tcsin(u)+ma^2b} \quad (5.8)$$

We have that the value of $ma^2b \gg 2tcsin(u)$, as for our single electron the value of the left side is roughly 10^{209}, while assuming a wavepacket of Planck-scale width, c is only roughly 10^{-54}. Thus, the velocity of the wavepacket is very nearly given as

$$\frac{lcos(u)}{ma^2b} = \frac{Re(z)}{|z|} \quad (5.9)$$

Using the aforementioned value for l. We note that (5.8) reduces to the standard formula for the velocity, in terms of the momentum, when the path discretization is real and the dual spatial situation is considered. For real discretization, (5.9) yields the unitless temporal speed of 1. However, when complex discretization is introduced, (5.9) leads to temporal speeds less than one. When the same initial wavefunction ((5.2) at $t = 0$) with the known value for l is allowed to evolve in some parallel universe, its movement through external time will generally be slower (relative to internal time) than in our own universe. Thus, each universe has a different rate of comparative temporal flow. Thus, allowing the particle to "quantum jump" to another universe would see its speed through time, compared to our own universe, decrease significantly by (5.9). Exploiting this property allows considerable potential applications.

VI. Superluminal Travel and Communication

The temporal speed factor (5.9) presents considerable ramifications on the nature and structure of this multiverse. Namely, a particle starting with the same initial temporal wavefunction will exhibit a lower temporal speed in the complex-discretized universes, compared to our own real-discretized universe. Thus, the single particle of the model progresses slower through external time in the alternate universes, compared to our own reality. This differential in temporal speed could pose considerable application, given a particular relationship between the relevant universes.

Namely, if the alternate realities of this combined theory exist "parallel" or "coexistent" in some sense, so that the future temporal barrier of one is smoothly topologically identified with the past barrier of the next, accessing these other realities might be possible. Such a scheme should assign the temporal potential barrier, being the border of two realities, the physical laws corresponding to one or the other. For example, the laws applying within each temporal barrier might correspond to the universe immediately preceeding it, in external time. The general principle, however, remains intact; we are engineering the external-time continuum so that successive universes in external time are parallel or coincident with each other, the temporal barrier linking external-time past and future serving as a way to topologically identify the future of one such universe with the past of the next. This programme follows the same rough principle as Roger Penrose's Cyclical Cosmology [1].

If accessing such parallel universes is a possibility, through a wormhole, space-time configuration, or some other contrivance, then the temporal factor of (5.9) could be used to achieve effective superluminal travel and communication. The continuum of external time, in accordance with the interpretation introduced in Section 1.B, would serve as the metric to determine the interaction points between such universes; namely, if an observer O leaves our universe U and enters a parallel universe U', at external time E in our universe and E' in U', and waits for an external time interval Y measured in U', then leaves, he will exit at external time E'+Y in U' and E+Y in U. That is, the total interval of external time traversed between entering and exiting the alternate universe is consistent with the frames of reference adopted in each separate universe. This is simply to say that, much as the most natural system would see spatial points in each separate universe "line up" in an analogous manner, so to do external time coordinates "line up," the that the total interval between entry and exit is the same in both realities. This is consistent with the general interpretation of external time serving as the metric for indexing the interaction between separate systems.

However, by (5.9) the same is not true of internal time. As the particle travels slower through external time in these other realities, the total interval of internal time experienced by the particle, between the external-time points corresponding to trans-universal entry and exit, is greater than the corresponding external-time in our universe. Such a journey between universes is consistent with the one-electron model we have been using, the movement of an apparatus from one universe to another simply being the corresponding movement of each of its constituent particles, each being distinct internal-time states of the one particle. As internal time determines the evolution of the particle's state, by (1.14), a starship transported into a parallel reality could accelerate to approach the speed of light, the latter determined by the ratio between spatial distance and internal time. However, due to the temporal differential of (5.9), the effective spatial velocity relative to external time could be very much greater, proportional to the factor $\frac{Re(z)}{|z|}$. Hence, effective superluminal travel is possible in this model.

The notion of "hyperspace" is a trope well-explored in the context of science-fiction, where superluminal travel is possible in some alternate or parallel dimension. Generally, this dimension, "hyperspace," is constituted so that small spatial distances in hyperspace correspond to comparatively large distances in "real space." Thus, the starship enters hyperspace, traverses a relatively short distance, and exits hyperspace at the point corresponding to its destination in "real space," thus completing a journey in effectively superluminal time [3]. The present model is similar, with these complex-discretized dimensions, "phasespace," offering conduits for superluminal travel. Here, however, the mechanism is different; rather than spatial distance being the relevant factor, temporal intervals are. The ship enters phasespace, makes a journey to a distant star at, say 50% light speed, taking 100 years in phasespace; when the ship re-enters "real space," however, only 10 years will have passed, allowing the ship to effectively travel at five times the speed of light. Use of constant acceleration might take advantage of time dilation, so that even the time measured within the ship might be comparatively small to that measured by the stationary observer in phasespace; however, consideration of this latter point would require combining quantum temporal dynamics and complex-discretization with relativistic quantum mechanics, which has not, as yet, been done.

VII. Conclusion

Work completed in [4] was developed from the path-integral framework, rather than assuming a generalized Schrodinger equation ad hoc. The one-electron model of quantum temporal dynamics, where a two-time theory provides a quantum model for time's passage and the reflections of a single particle between two barriers produce all the particles seen in the universe, was combined with the multiphase discretization metaverse, where complex-discretization of functional integrals produces a panoply of parallel universes. The temporal cycles emerging from quantum temporal dynamics were postulated to correspond to the alternate realities of MDM. By using the new path-integral framework from QTD, complex-discretization was built into MDM, offering the laws of physics applying to these separate realities. Some conclusions were drawn from these results, including the likely sizes of certain constants of the theory, the sizes of the temporal barriers, and the differential in temporal flow between universes.

As a result of this last property, we discovered that superluminal travel would be possible if these alternate realities could be accessed. Given that a certain topological identification exists so that the temporal cycles are coincident in external time, so that these universes can indeed be accessed, travel through an alternate dimension could proceed at a speed greater than that of light. This theory, however, is nonrelativistic, and the development of a complex-discretized quantum field theory, as well as a relativistic version of QTD, could allow these results to be extended significantly.

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